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SOVIET PHYSICS

"Doklady"

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SOVIET PHYSICS

"Doklady"

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A year ago the American Institute of Physics established "Soviet Physics - JETP" as its first step in bringing before American physicists information about the scientific activities of Soviet physicists. The demand for translations of other Russian physics journals has encouraged the Institute to broaden its program. "Soviet Physics - Doklady" and the other translation journals listed on the back cover of this issue constitute the expanded program of the Institute.

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Doklady

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THE THEORY OF MAGNETOSTRICTION OF NICKEL SINGLE CRYSTALS

N. S. Akulov, Act. Memb. Acad. Sci. Belorussian SSR

1. After the discovery of the anisotropy law [1], which enables us to calculate the magnetostriction of Fe and Ni single crystals in the saturation region, we attacked the problem of calculating magnetostriction, as well as other exchange effects, as a function of the magnetization [2, 3]. In the case of crystals with six directions of easy magnetization (Fe, etc.) two methods were proposed for the solution of this problem [2, 3]. The first method was based on the fact that in the case of longitudinal inversion, i.e., where there is a 180° displacement of the boundaries between domains of spontaneous magnetization, magnetostriction does not occur and no work is done to deform the crystal in the course of magnetization. The situation is otherwise when a 90° boundary shift occurs (transverse inversion). In this case magnetostriction occurs in those regions of the crystal where this boundary displacement takes place. As a result additional work may be done against the internal strains of the crystal. It follows that in weak fields longitudinal inversion takes place, and only with increasing field strength does transverse inversion occur. The magnetostriction curves of Fe single crystals, based on complete shifting of boundaries in a succession of such longitudinal and transverse inversions, have agreed quite well with experiment.

Shortly after the publication of the above work [2] Heisenberg [3] proposed a statistical method of calculating the sequence of processes of longitudinal and transverse inversions. This method is of a formal nature and is based on the calculation of the distribution of domains of a crystal, for a given intensity of magnetization, at which the statistical probability of the distribution is a maximum. The experimental curves obtained by Webster as well as by Honda and his collaborators lie between the theoretical curves of Akulov and of Heisenberg.

In 1933 Gans and his collaborators attempted to calculate the magnetostriction curves of crystals with 8 axes of easy magnetization (Ni, etc.). Gans used Heisenberg's method. His calculations gave for $H \parallel [110]$ curves which were in qualitative agreement with experiment. However for $H \parallel [111]$ the discrepancies went far beyond the limits of experimental error (see Fig. 2).

Thus the problem of magnetostriction of single crystals of the above type has, despite its importance, remained unsolved up to the present time.

2. Let us begin by considering the case in which the field is directed along a diagonal axis $H \parallel [110]$. Calculation of the inversion is very simple here, since there is only one kind of longitudinal and one kind of transverse inversion. Consequently the following simple scheme is correct in this instance: first longitudinal, next transverse inversion, and, finally, rotation from the axis of easy magnetization which is closest to the external field. In this case the longitudinal inversion is completed at

$$I_A = \frac{1}{2\sqrt{2}} I_r. \quad (1)$$

Transverse inversion is concluded at

$$I_A = \frac{\sqrt{2}}{\sqrt{3}} I_r. \quad (2)$$

The magnitude of the magnetostriction is calculated as in [2] on the basis of the anisotropy law. The result is

$$\lambda = 1/2 \lambda_{111}. \quad (3)$$

It is easy to calculate the additional terms which are obtained if we take into consideration higher order terms in the anisotropy tensor. In the present instance they do not result in any significant change.

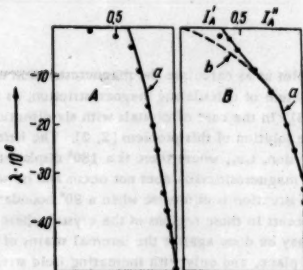


Fig. 1. Theoretical curves: a) Akulov; b) Heisenberg. Ni. A) $H \parallel [111]$; B) $H \parallel [110]$.

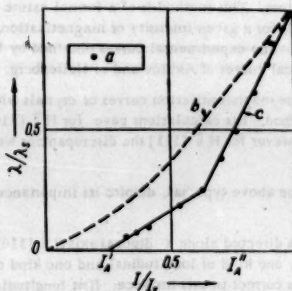


Fig. 2. $H \parallel [111]$. a) Experimental; b) Heisenberg theory; c) Akulov theory.

$2 \rightarrow 4$ take place after the conclusion of the transition $1 \rightarrow 4$, i.e. in stronger fields. These transitions $2 \rightarrow 3$ and $2 \rightarrow 4$ occur as a result of the change-over of those same domains which were in state 2. Thus an interpenetration of processes $2 \rightarrow 3$ and $2 \rightarrow 4$ takes place. In other words, we have coupled transitions of the type



(5)

For generality it is advisable to allow for the fact that the probability of the transition $2 \rightarrow 3$ may differ from the probability of the transition $2 \rightarrow 4$. Nevertheless, as we shall see, even the simplest condition $\alpha = 1 - \alpha = 1/2$ results in good agreement with experiment. After the conclusion of the indicated "coupled" transitions the process of magnetization is due to transitions $3 \rightarrow 4$. These are accompanied by twice the

After the second anisotropic point the process of rotation begins, which in both our theory and Heisenberg's theory is calculated alike. Details of the calculation are given in reference [2].

In Fig. 1 we have compared the results of calculations by our method and by Heisenberg's method. There are appreciable differences in the region of inversion for a digonal axis. The experiments of Masyama give results which are in considerably better agreement with our curve than with the curve of Gans, which had been calculated using Heisenberg's method.

3. We shall now consider the case in which the external magnetic field is parallel to an axis of easy magnetization, i.e. (in the case of Ni crystals) along a trigonal axis.

The first type of longitudinal inversion is a 180° boundary displacement. This gives the maximum reduction of free energy of the external field in comparison with those displacements which differ from 180° . In the initial state of the crystal $1/8$ of the volume of the crystal is magnetized along one easy axis of magnetization, $1/8$ along another, etc. After conclusion of the longitudinal inversion the crystal will have the magnetization

$$I_A' = 1/4 I_s. \quad (4)$$

This inversion is due to transitions of spins which originally were oriented in the direction $4 \rightarrow 1$, but after the transition were oriented $1 \rightarrow 4$ (see Fig. 3).

The transition of spins from position 2 to 3 (see Fig. 3) is also accompanied by a 180° boundary displacement. However the projection of the external field on the line of the displacement is considerably smaller here. Therefore the transitions $2 \rightarrow 3$ and

$2 \rightarrow 4$ take place after the conclusion of the transition $1 \rightarrow 4$, i.e. in stronger fields. These transitions $2 \rightarrow 3$ and $2 \rightarrow 4$ occur as a result of the change-over of those same domains which were in state 2. Thus an interpenetration of processes $2 \rightarrow 3$ and $2 \rightarrow 4$ takes place. In other words, we have coupled transitions

magnetostriction of the transitions (5) and the decrease of free energy of the external field is considerably smaller.

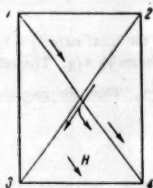


Fig. 3.

We now calculate the intensity of magnetization I'_A after conclusion of the coupled transitions. It is easily seen that:

$$I'_A = I_A + 3I_s \left[2 \frac{1-\alpha}{z} \cos \varphi + \frac{\alpha}{z} \cos \varphi + \frac{\alpha}{z} \right] \quad (6)$$

Here the coefficient 3 corresponds to the number of trigonal axes of the crystal, forming the angle

$$\varphi = \arccos \frac{1}{z}, \quad (7)$$

with the external magnetic field, which is directed along the fourth tetragonal axis; z is the number of directions of easy magnetization.

The quantity $2 \frac{1-\alpha}{z} \cos \varphi$ is the increase of the projection of the resulting magnetic domain spins in the transition from state 2 to state 3. In the transition from state 2 to 4 this projection acquires the additional increase $\frac{\alpha}{z} \cos \varphi + \frac{\alpha}{z}$. On the basis of (4) and (7) with coupled transitions ($\alpha = 1/2$) we find from (6)

$$I'_A = 3/8 I_s. \quad (8)$$

It is now easy to calculate the third portion of the magnetization curve, which comprises the range due to the transitions $3 \rightarrow 4$.

The fraction of the crystal volume that is magnetized in state 3 at the end of the second stage is $\frac{1}{z} + \frac{1-\alpha}{z}$. The relative change of the projection of the resultant spin in the $3 \rightarrow 4$ transition is given by the difference $1 - \cos \varphi$. Thus the additional magnetization resulting from the transitions $3 \rightarrow 4$ is

$$I_s - I'_A = 3I_s \frac{2-\alpha}{z} (1 - \cos \varphi), \quad (9)$$

which for $\alpha = 1/2$ agrees with (8).

In each of the three ranges of magnetization the magnetostriction will depend linearly on the intensity of magnetization, as transitions which are identical in nature induce an identical increase of the exchange effect.

It is therefore necessary to determine the ordinates of the special points of anisotropy which determine the beginning and end of the three ranges of magnetization. The abscissas are determined by the quantities Γ_A and I'_A .

In accordance with the anisotropy law the magnetostriction is defined by the relation [1]

$$\lambda = a_0 + a_1 (s_1^2 g_1^2 + s_2^2 g_2^2 + s_3^2 g_3^2) + 2a_2 (s_1 s_2 g_1 g_2 + s_2 s_3 g_2 g_3 + s_3 s_1 g_3 g_1), \quad (10)$$

where

$$a_0 = -1/8 \lambda_{100}, \quad a_1 = 1/8 \lambda_{100}, \quad a_2 = 1/8 \lambda_{111}. \quad (11)$$

We note incidentally that (11) has sometimes been ascribed incorrectly to Becker, although it was actually obtained for the first time in [2] (equation (10) was obtained in [1]).

From (10) and (11) it follows that when H is oriented along a trigonal axis, i.e., when

$$g_1 = g_2 = g_3 = \frac{1}{\sqrt{3}}, \quad (12)$$

for transitions $1 \rightarrow m$ we have

$$\lambda' = \lambda_{111} [(s_1 s_2 + s_2 s_3 + s_3 s_1)_l - (s_1 s_2 + s_2 s_3 + s_3 s_1)_m]. \quad (13)$$

Hence for the transitions $1 \rightarrow 4$ and $2 \rightarrow 3$ we find

$$\lambda_A^* = 0. \quad (14)$$

For the transition $2 \rightarrow 4$ in the initial state $s_1 = s_2 = -s_3 = 1/\sqrt{3}$, and in the final state $s_1 = 1/\sqrt{3}$ ($i = 1, 2, 3$). Thus, according to (13), the increase of magnetostriction per unit volume is here $\frac{1}{3} \lambda_{111}$. The relative increase of volume of the crystal in which the $2 \rightarrow 4$ transitions take place is $\propto \frac{3}{2}$. Thus, for magnetostriction which is due to coupled transitions we obtain

$$\lambda_A^* = \frac{4\alpha}{3} \lambda_{111}. \quad (15)$$

For the subsequent process $3 \rightarrow 4$ we have analogously, in accordance with (15)

$$\lambda_{111} - \lambda_A^* = \frac{4}{3} (2 - \alpha) \lambda_{111}. \quad (16)$$

From (15), with $\alpha = 1/2$, there results

$$\lambda_A^* = 1/4 \lambda_{111}. \quad (17)$$

Connecting the obtained anisotropy points by straight lines, as shown above, we have for $\lambda(H)$ a broken curve which agrees very well with the results of Masiyama's measurements.

In Fig. 2 we also give Gans' curve which was calculated by Heisenberg's method on the basis of his formula

$$\lambda = \frac{\lambda_{111}}{2} \left(\sqrt{8 \frac{I_1}{I_2} + 1} - 1 \right). \quad (18)$$

It is easily seen that the Gans-Heisenberg equation disagrees completely with experiment, so that doubt is thrown on the theoretical justification for Heisenberg's method.

Thus, on the basis of our method of calculating the coupled reconstructions of magnetic domains, it has been possible for the first time to give a quantitative explanation of the magnetostriction curves of crystals with 8 easy axes of easy magnetization.

Received April 21, 1955

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THE EFFECT OF REABSORPTION ON THE BREADTH OF SPECTRAL LINES

L. M. Biberman and E. M. Novodvorskaia

(Presented by Academician G. S. Landsberg, July 8, 1955)

1. It is known that reabsorption of radiation by the source very strongly influences the shape of a spectral line. In some instances it is difficult to obtain reliable data on the entire shape of a line, while, at the same time it is comparatively easy to measure its breadth. In this connection it is of interest to know the relationship between the line breadth and parameters of the source.

In the simplest case, when the temperature is independent of the coordinates while the pressure is high enough to establish a state that is close to thermodynamical equilibrium, the intensity distribution I_ν of the line will be given by

$$I_\nu = B_\nu (1 - e^{-k_\nu l}), \quad (1)$$

where B_ν is the intensity of radiation of a black body for frequency ν ; k_ν is the absorption coefficient; l is the spatial extent of the source in the direction of observation.

Converting to the usual variables:

$$\omega = \frac{2(\nu - \nu_0)}{\Delta\nu_D} \sqrt{\ln 2},$$

where ν_0 is the frequency of the middle of the line, and $\Delta\nu_D$ is the Doppler broadening.

It is then easy to obtain from (1) an expression which defines the line breadth:

$$f(\omega_{1/2}) = \frac{1}{k_0} \ln \frac{2}{1 + e^{-k_0 l f(\omega)}}, \quad (2)$$

where k_0 is the absorption coefficient at the middle of the line in the case of pure Doppler broadening.

In the case of simultaneous Doppler and collision broadening

$$f(\omega) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{a^2 + (\omega - y)^2}, \quad \text{where } a = \frac{\Delta\nu_c}{\Delta\nu_D} \sqrt{\ln 2}, \quad (3)$$

where $\Delta\nu_c$ is the line breadth due to collision broadening alone (the breadth which is due to natural broadening is added to $\Delta\nu_c$).

The curves in Fig. 1 represent the dependence of the breadth of a spectral line, expressed in units of ω , on the optical density for different values of a . These curves were obtained by using (2) and tables for (3) in a not too complicated calculation. In Fig. 2 the initial portions of these curves are given on an increased scale. The limiting case of pure Doppler broadening ($a = 0$) was observed previously by Burger and Clitter [1]. It is noteworthy that a considerable range shows linear dependence of $\omega_{1/2}$ on $k_0 l$. This is true for all values of a .

For comparatively large optical densities it is possible to obtain an approximate formula for the functional relationship of $\omega_{1/2}$ and $k_0 l$.

$$\omega_{1/2} = \sqrt{\frac{ak_0 l}{\pi^{1/2} \ln \frac{2}{1 + e^{-k_0 l/(a)}}} - a^2}, \quad (4)$$

where $f(0) = e^{a^2} [1 - \Phi(a)]$, where $\Phi(a)$ is the probability function.

The curves obtained by the use of (4) are also shown in Fig. 1. The region of the applicability of (4) for small values of $k_0 l$ increases as a becomes larger. From (2) we computed $\omega_{1/2}$ as a function of $k_0 l$ for $a = 10$. This case is not shown in Fig. 1, as the results of the calculation are quite accurately described by (4).

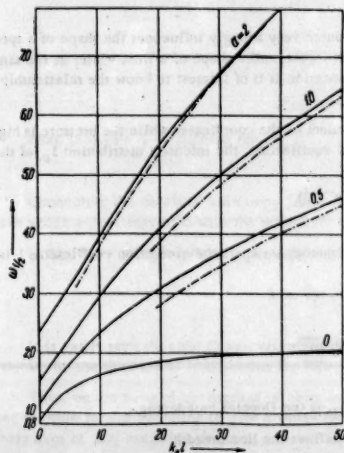


Fig. 1. Spectral line breadth as a function of optical density. Continuous curves were calculated by using equations (2) and (3); broken curves were calculated by using (4).

ness of the radiating layer in [2] to be ~ 0.5 cm, it is easily established that the broadening induced by the resonance interaction is smaller than the broadening observed in [2] by a factor of the fourth or fifth order of magnitude.

Thus the explanation given by Viatskin and Nagibina is unsuitable. On the other hand, a calculation of the reabsorption is quite adequate to explain the results obtained in [2].

In [2] it was noted that the departure of the calibration curves from linearity is observed for the same values of the concentration of the admixture as is the beginning of the increase of line breadth. This fact was not explained in [2]. By showing the decisive influence of reabsorption on the change of line breadth we at the same time explain the above-mentioned coincidence, as it is well known that the departure of the curves of total absorption from linearity is also induced by reabsorption [4].

3. The presence of linear increase portions in the curves of Fig. 1 enables us to suggest a means of determining the concentration of emitting atoms in the source. For this purpose it is necessary to determine $\Delta\lambda/\Delta c\%$ from experimental relationships, where $\Delta\lambda$ is the increase of line breadth measured in units of length which corresponds to the change in the per cent concentration expressed in arbitrary units $\Delta c\%$. Having found the Doppler broadening $\Delta\lambda_D$, we must then calculate $\Delta\omega_{1/2}/\Delta c\%$ and equate this value to the theoretical value of $\Delta\omega_{1/2}/k_0 l$, which enables us to obtain the optical density. If the probability of the optical transition is known, the true concentration is found in the usual way from the obtained value of $k_0 l$.

2. A. Ia. Viatskin and I. M. Nagibina [2] measured the shapes of lines in arc emission as a function of the concentration of admixtures expressed as per cent. For the three lines of different elements which they investigated they obtained the same kind of functional dependence of line breadth on concentration, divided into a small region of unchanging breadth and a fairly large region of linear increase. Thus the results of [2] correspond to the curves shown in Figs. 1 and 2.

Viatskin and Nagibina attempted to explain the observed linear increase of line breadth in arc spectra as being due to broadening of the emitted line of the individual atom as a result of resonance interaction with atoms of the same element. It can easily be shown that under the conditions of the experiments performed by these authors resonance broadening could not be responsible for the observed effects.

Indeed, by using the well-known expression for the line breadth due to resonance interaction [5], a_r , and also the expression for k_0 , it is easy to obtain

$$a_r = \frac{2}{3\pi^{1/2}} k_0 \lambda_0 \quad (\lambda_0 \nu_0 = c).$$

Evaluating the optical density $k_0 l$ by comparing the calibration graphs given in [2] with the theoretical curves for total absorption [3] and assuming the thick-

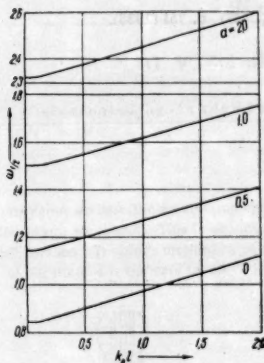


Fig. 2. Spectral line breadth as a function of optical density for small values of the argument.

TABLE 1

Line	$\Delta \lambda_D, \text{\AA}$	$\frac{\Delta \lambda}{\Delta c \%}$	$\frac{\Delta \omega_{1/2}}{\Delta c \%}$	$\frac{\Delta \omega_{1/2}}{\Delta k_0 l}$	$k_0 l$ corresponding to $c = 1\%$
Mu 2949 A	0.02	$5.0 \cdot 10^{-4}$	$2.17 \cdot 10^{-2}$	0.15	0.14
Pb 2873 A	0.01	$6.7 \cdot 10^{-4}$	$5.55 \cdot 10^{-2}$	0.15	0.37
Cd 3261 A	0.016 *	$11 \cdot 10^{-4}$	$5.8 \cdot 10^{-2}$	0.15	0.39

In [1] instead of 0.016 the figure 0.014 is given, which corresponds to the temperature 4100° K instead of 5100° K as given. The Doppler broadening of the remaining two lines in [2] is calculated on the basis of 5100° K.

obtained for values of a of the order of unity, which is quite natural under the conditions of [2]. Similar values of a have been observed under similar conditions by other experimenters [4].

The results obtained for the optical densities agree in order of magnitude with the results of calibration by the usual method — calibration by graphs and curves of total absorption. Complete agreement would only be accidental, since both methods fail to allow for a number of factors (unresolved hyperfine structure, spatial non-uniformity of the luminous cloud, etc.).

If the concentration of emitting atoms in the arc is changed as a result of a proportional change of the concentration of the element in the electrode, we obtain a relationship between these two quantities. Ordinarily a linear relationship is assumed. Experimental data [2] in conjunction with the above theoretical considerations may be considered a confirmation of this assumption. It is important that the slope of the linear portion of the theoretical curves of $\Delta \omega_{1/2} / \Delta k_0 l$ is practically independent of a (the value is ~ 0.15). This circumstance eliminates the problem of allowing for an instrumental effect, since instrumental broadening reduces as a rule to a change of the value of a which does not affect the result of the calibration.

As an example we give in Table 1 the concentration calibration used in [2].

From knowledge of the optical density $k_0 l$ and the value of a we can evaluate the concentration of the emitting atoms. Thus the greatest concentration of cadmium in [2] was $\sim 10^{15} \text{ 1/cm}^3$ (assuming that the thickness of the emitting layer was $\sim 0.5 \text{ cm}$).

A calibration carried out with a suitable choice of the parameter a makes it possible to obtain practically complete coincidence of the theoretical and experimental curves. This agreement is not surprising, since by a choice of two parameters it is always possible to make two straight lines coincide. It is significant, however, that quantitative agreement is

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AN ELECTRON OPTICAL METHOD OF STUDYING THE DYNAMICS OF THERMAL PROCESSES IN OPAQUE MEDIA

G. V. Spivak and T. N. Dombrovskaya

(Presented by Academician A. V. Shubnikov, June 14, 1955)

In this paper we describe an electron optical method for the visual observation of temperature wave front propagation in opaque media. This is an electron optical "generalization" for opaque media of Toepler's "Schlieren" method. Toepler's method is, of course, applicable only to optically transparent media. An optical analog of our method is the case in which in addition to the lens the light source would be modulated by optical inhomogeneities.

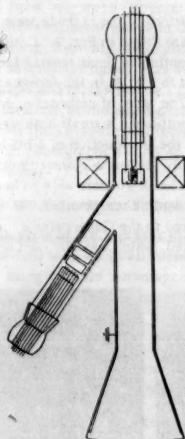


Fig. 1. Design of the instrument for visualization of temperature waves.

The proposed method is based on the following:

- 1) It is possible to obtain contrast in an electron optical image, as was shown previously [1], by including magnetic inhomogeneities in the surface of a flat sample used as the cathode; and 2) ferromagnetic materials exhibit a marked change of their properties at the Curie point.

The instrument used for visualization of the thermal processes was a model of a low-power secondary emission microscope with magnetic focusing (see Fig. 1). The main peculiarity of the instrument is the construction of the cathode. At the cathode various means were used to establish and strengthen the electron optical contrast, which was modulated by a thermal process propagated in the solid material.

The object under investigation, which served as a secondary emission cathode, was bombarded by low speed primary electrons. The secondary electrons were accelerated to a few kilovolts and focused on the fluorescent screen by an ironclad magnetic lens that at the same time magnetized the object, which contained magnetic inclusions. The aquadag coating served as anode.

The mechanism of electron optical visualization of a thermal process in a specimen is based on the fact that the electron density at the object cathode is modulated by the local magnetic field.

Let us picture (see Fig. 2) the magnetic inhomogeneities at the cathode (indicated by shading) as separated by nonmagnetic material (unshaded). The electron density is modulated by the local magnetic fields as the trajectories spiral around the magnetic lines of force.

When the plane f_m is focused on the fluorescent screen, region A* will be brighter than region B*. Consequently, the bright spots form a picture of the nonmagnetic portions, while the dark spots correspond to the ferromagnetic material. When within the solid material there is a source of heat due to change of magnetization that has been caused by heating, it is possible to observe the advance of the thermal process on the face of the cathode. The construction of the cathode to provide the required contrast and heating to the Curie point and higher is shown in Fig. 3.

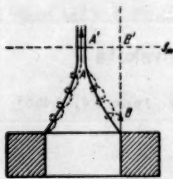


Fig. 2. Mechanism of the formation of local magnetic contrast.

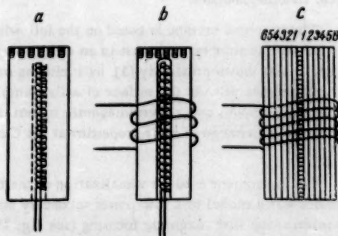


Fig. 3. Construction of the cathode for stronger contrast.

We shall consider two cases of visualization of the temperature wave front: in a magnetically homogeneous medium and in a magnetically inhomogeneous material. The inhomogeneity of the material may be natural or artificial. As a result of the high thermal conductivity of ferromagnetic materials, when the temperature wave front is not steep the contrast in the first case is ordinarily insufficient for observation of the spreading of the process. Only for a fairly steep front is it possible to fix the position of the thermal wave front in a homogeneous material. Improved contrast for observation of the thermal wave front with gradual increase of temperature was achieved by several methods corresponding to the second case.

Four variations of the cathode were tried, three of which are shown in Fig. 3. Cathodes a and b have discontinuous nonmagnetic inclusions (copper) (shaded in Fig. 3) in the ferromagnetic (pure nickel). The faces of cathodes a, b and c were carefully polished. A small hole was bored into a and b for the introduction of a bifilar tungsten helix. By sending a current through the inside or outside helix we were able to heat the cathode to the required temperature. When the sample was heated to the Curie point a magnetic transition occurred, which was observed on the screen.

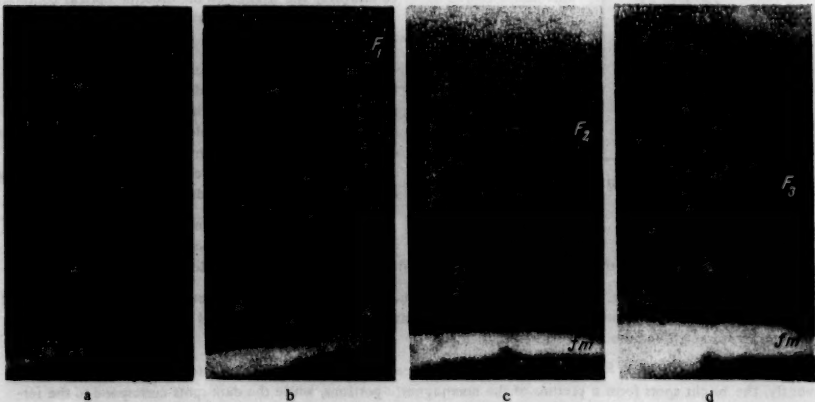


Fig. 4. Advance of the temperature wave front in an optically opaque medium. a, b, c and d show the position of the wave front relative to the band f_m , which indicates the plane of magnetic inhomogeneities. F_1 is the first position of the temperature wave front; F_2 and F_3 are later positions.

In the case of type *b*, it was possible to first use one of the helices to heat the cathode to a temperature close to the Curie point, so that the second helix could then drive the temperature quickly above the Curie point. This design ensured high sensitivity of the system, increasing the speed of the transition of the ferromagnetic to a paramagnetic material by lowering the magnetic susceptibility of the specimen.

For the gradual temperature wave front, the contrast was obtained through periodicity of the nonmagnetic inclusions. This, however, had certain disadvantages, so that a cathode of type *c* was prepared. This cathode was made of strips of nickel 0.2 mm thick with progressively increasing admixtures of silicon (0.5; 1; 2; 3; 4 %).

In order to make it possible to adjust the instrument for plane f_m of the magnetic nonuniformities, one diamagnetic inhomogeneity was inserted into the cathode, in the form of a copper strip (shaded in Fig. 3). This cathode possessed a practically continuously distributed magnetic inhomogeneity. The strips with larger silicon content possessed a lower Curie point and were accordingly placed closer to the source of heat. This resulted in improved contrast in the picture of the wave front as a result of stronger temperature dependence of the jump of the electron-optical refractive index.

A fourth type of cathode was made of a ferrite. Due to its smaller thermal conductivity compared with nickel, for a non-steep temperature wave front there was insufficient contrast for observation of the boundary. Fig. 4 shows different stages in the advance of the temperature wave front on the face of a cathode of the type shown in Fig. 3c. Fig. 4a depicts the geometry of the surface of a cold cathode. In Fig. 4b the cathode was heated and the instrument was adjusted for plane f_m , which is seen as the white band in the lower part of the photograph and represents the copper inclusion. In the same photograph there can be seen the increasing lightening of the field in the upper part, although a white band has not yet been formed. This lightening of the field represents the advancing temperature wave; the boundary of the lightened region is the front of the temperature wave. In photographs 4c and 4d we see the front approaching a geometric inhomogeneity (the dark spot in the photograph).

When the heat was turned off it was possible to observe the advance of the front in the reverse direction.

In view of the fact that the Curie point in different materials varies from tens of degrees to a value of the order of 10^3 , we may in all these cases investigate the dynamics of the thermal process by the method described above, using suitable ferromagnetic coating of the nonferromagnetic material being studied (electron-optical "staining" - analogous to staining in the optical case in the preparation of transparent objects for a light microscope).

Thus we have shown the possibility of visualizing the behavior of a ferromagnetic material in the neighborhood of the Curie point, its transition to a paramagnetic state, and observation of the structure and velocity of propagation of the temperature wave in opaque media.

Measurement of the temperature contrast of a "magnetic" image gives quantitative data concerning the course of magnetic and thermal processes, which is of value, for example, in determining the thermal parameters of a specimen that are associated with sudden rapidly passing phenomena.

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FLOW DEFORMATION OF MACROMOLECULES AND ITS EFFECT ON THE SIGN OF THE DYNAMIC BIREFRINGENCE OF A POLYMER IN SOLUTION

E. V. Frisman and V. N. Tsvetkov

(Presented by Academician A. A. Lebedev July 9, 1955)

In order to explain the optical behavior of a polymer solution we proposed an optical model [1] of a macromolecule in solution in the form of a molecular ball of ellipsoidal shape which strongly immobilizes the solvent. The average refractive index n_0 of such an ellipsoid is defined by the relation

$$n_0^2 - n_s^2 = \frac{1}{\omega} (n_k^2 - n_s^2). \quad (1)$$

Here n_k and n_s are the refractive index of the dry polymer and of the solvent, respectively; ω is the "volumicity" of the polymer in solution. Here $\omega = \frac{v}{m} \rho = \frac{v \rho N_A}{M}$, where ρ is the density of the polymer, M is its molecular weight, m is the mass of the molecular ellipsoid, v is its volume, and N_A is Avogadro's number.

According to the statistical theory [2], in the absence of external forces a molecular ball in a solution possesses internal optical anisotropy which is dependent on the difference of the principal polarizabilities of a segment in vacuum $\alpha_1 - \alpha_2$.

By our hypothesis the optical behavior of a macromolecule in solution is determined not only by its internal anisotropy but also by the anisotropy of its shape. Applying to this model the laws of colloidal optics [3-5], we obtained [6] an expression for the total difference of the polarizabilities of a molecular ball in solution.

The dynamic double refraction of light by a polymer solution is the result of the intrinsic anisotropy of the undeformed ball, its asymmetrical shape and the anisotropy resulting from the deformation of the ball in the flow. If it is assumed that under the condition $g \rightarrow 0$ (where g is the velocity gradient of the flow) birefringence is determined principally by the orientation of asymmetric and anisotropic balls, then by employing the theory of birefringence, in a flow, of a suspension of rigid ellipsoidal particles [8], and also the expression for the difference of polarizabilities of a molecular ball [6], one can obtain an expression for the dynamic optical constant $[n]$:

$$[n] = \left(\frac{\Delta n}{g c \eta_0} \right)_{g \rightarrow 0} = [n]_e + [n]_f. \quad (2)$$

Here Δn is the magnitude of the double refraction; c is the concentration of the solution (g/cm^3); η_0 is the viscosity of the solvent. Part of the dynamic optical constant $[n]_e$ is due to the intrinsic anisotropy of the molecular ellipsoid, and part $[n]_f$ is due to the anisotropy of its shape.

Dividing both sides of equation (2) by the characteristic viscosity η we obtain [6]:

$$\frac{[n]}{[\eta]} = 0.11 \frac{b}{kT} \frac{(n_s^2 + 2)^3}{n_s} (\alpha_1 - \alpha_2) + \frac{1}{30 \pi \rho^2 N_A kT} \frac{(n_k^2 - n_s^2)^3}{n_s^2} (L_2 - L_1) b \frac{M^2}{v}. \quad (3)$$

* This result was obtained independently by Peterlin and Kopic [7].

Here k is the Boltzmann constant; T is the temperature; R is the gas constant; L_1 and L_2 are constants determined by the eccentricity of the ellipsoid; b is a factor determined by the axis ratio p : $b = (p^2 - 1) / (p^2 + 1)$.

The orientation theory of the dynamic optical properties of solutions of chain molecules [9, 10], taking into consideration only the intrinsic anisotropy of the balls, gives an expression for $[n]/[\eta]$ containing only one term, which differs from the first term of (3) only in its numerical coefficient. Since the anisotropy $\alpha_1 - \alpha_2$ of a segment is identical for different fractions in a given solvent, according to Kuhn the quantity $[n]/[\eta]$ should not depend on the molecular weight of the polymer.

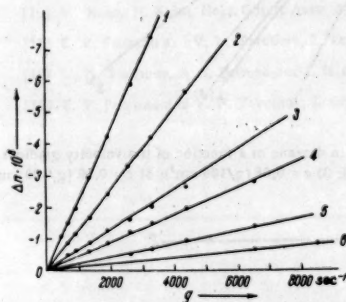


Fig. 1. Birefringence of solutions of polystyrenes ($M \approx 3 \cdot 10^5$) in dioxane as a function of the velocity gradient of the flow. 1) $c = 2.0$ (g/100 cm³); 2) $c = 1.5$ (g/100 cm³); 3) $c = 1.0$ (g/100 cm³); 4) $c = 0.8$ (g/100 cm³); 5) $c = 0.5$ (g/100 cm³); 6) $c = 0.3$ (g/100 cm³).

Allowance for the effect of the shape results in an increase in the value of $[n]/[\eta]$ with increased molecular weight M [Equation (3)]. Experimental data [11, 12] indicate constancy of $[n]/[\eta]$ for different molecular weights when the role of the shape is insignificant ($n_k \approx n_s$). If $n_k - n_s$ is large a sharp rise of the curve $[n]/[\eta] = f(M)$ is observed. This result was obtained [11] for solutions of polystyrene in butanone ($n_k - n_s = 0.22$), where the birefringence of the solutions of all the fractions investigated has a positive sign.

The experimental idea of the present paper is essentially as follows. In solutions of polystyrene $[n]_e$ always has a negative value, whereas $[n]_f$ is always positive. The sign of the measured dynamic optical constant consequently depends on the ratio of these quantities. Since with increased molecular weight the role of the shape increases, it may be expected that with a suitable choice of solvent the dynamical constant will have a different sign for different molecular weights. With this in mind we measured the dynamic birefringence of solutions of two fractions of polystyrene ($M_A \approx 3 \cdot 10^5$; $M_B \approx 8 \cdot 10^5$) in dioxane. For this system $n_k - n_s = 0.18$.

Our instrumental setup and versatile dynamic optometer have been described previously [11, 13].

For the study of solutions of fraction A we used a rotor ($l = 5.5$ cm), with the opening $\Delta r = 0.036$ cm. Solutions of fraction B were investigated with a device of the same design as the other but differing in the length of the operating portion, with $l = 10$ cm and with the opening $\Delta r = 0.085$ cm.

The results of the measurements, which are given in Fig. 1, show that for solutions of fraction A, Δn increases proportionally to g in the entire range of velocity gradients which was investigated. The birefringence of solutions of this fraction is negative.

Fig. 2 shows the function $\Delta n = f(g)$ for different concentrations of solutions of fraction B in dioxane. The curves are quite unusual. For small velocity gradients the birefringence is positive and increases proportionally to the velocity gradient g . As g increases the proportionality between Δn and g disappears, the curve $\Delta n = f(g)$ reaches its maximum and then passes into the region of negative values. Further increase of the velocity gradient leads to a marked increase of negative birefringence.

Decrease and change of sign of birefringence results from the appearance of a negative photoelastic effect, whose role increases with a rise of the velocity gradient (hydrodynamic force). The smaller the concentration of the solution (smaller viscosity) the larger the values of g at which the birefringence vanishes.

Thus for low velocity gradients solutions of the fraction of high molecular weight exhibit positive birefringence, and solutions of the lower molecular weight exhibit negative birefringence. These data are convincing evidence that the shape effect depends on the molecular weight and is determined by the molecular ball as a whole.

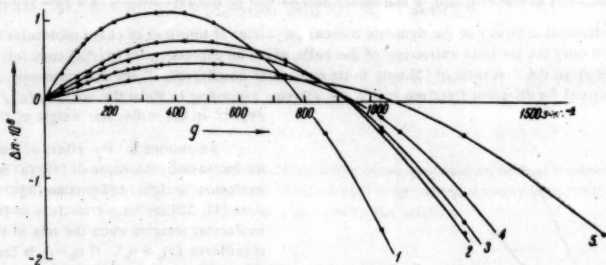


Fig. 2. Birefringence of solutions of polystyrene ($M \approx 8 \cdot 10^5$) in dioxane as a function of the velocity gradient of the flow. 1) $c = 0.20$ (g/100 cm³); 2) $c = 0.10$ (g/100 cm³); 3) $c = 0.08$ (g/100 cm³); 4) $c = 0.06$ (g/100 cm³); 5) $c = 0.04$ (g/100 cm³).

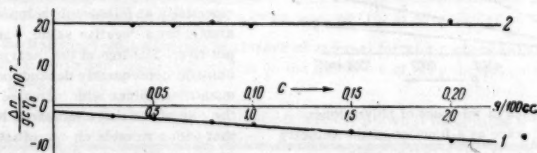


Fig. 3. The function $(\Delta n/gc\eta_0)g \rightarrow 0 = f(c)$ for solutions of polystyrene in dioxane, 1) $M \approx 3 \cdot 10^5$; 2) $M \approx 8 \cdot 10^5$.

The value of the dynamic optical constant $[\eta]$ for solutions of fraction A, which was obtained by extrapolating the curve $(\Delta n/gc\eta_0)g \rightarrow 0 = f(c)$ for $c \rightarrow 0$ (see Fig. 3), is $[\eta] = -0.16 \cdot 10^{-7}$. For solutions of fraction B the quantity $(\Delta n/gc\eta_0)g \rightarrow 0$ in the investigated range of concentrations is practically independent of the concentration and has the value $[\eta] = +21 \cdot 10^{-7}$.

In accordance with (3) the ratio $[\eta]/[\eta]$ shows strong dependence on the molecular weight: for fraction A, $[\eta]/[\eta] = -1.86 \cdot 10^{-10}$; for fraction B, $[\eta]/[\eta] = +23 \cdot 10^{-10}$.

There is no doubt that the experimental data enable us to draw the following general conclusion. A large positive value of the dynamic optical constant of solutions of fraction B indicates that for small velocity gradients the photoelastic effect plays an insignificant role in comparison with the shape effect; its importance increases, however, as the velocity gradient increases. This indicates that at $g \rightarrow 0$ the molecular ball must have considerable asymmetry of shape, which is the decisive factor for dynamic birefringence under these conditions.

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CROSS SECTIONS OF INELASTIC INTERACTIONS BETWEEN 120 AND 380 MEV NEUTRONS AND NUCLEI

V. I. Gol'danskii, A. A. Koval'skii, V. S. Pen'kina and E. Z. Tarumov

(Presented by Academician I. E. Tamm July 13, 1955)

For the description of the interaction of high-energy neutrons with nuclei extensive use has been made of the optical model, in which the properties of a nucleus are described by the refractive index $\bar{n}$ of the nuclear material and by the absorption coefficient K . Analyzing the first experiments [1] on the determination of the total nuclear cross sections for high energy neutrons (90 Mev), Serber [2] obtained a satisfactory description of the experimental data by using an optical model in which the nucleus is considered a degenerate Fermi gas. He determined K from n - p and n - n scattering cross sections (with a correction for the Pauli principle) taking the depth of the nuclear potential well to be $V \approx 30$ Mev (V is related to $\bar{n}$ by the relationship $\bar{n} = \sqrt{1 + V/E}$, where E is the energy of the bombarding neutron).

The results of measuring the inelastic interaction cross sections of 90 Mev neutrons [3, 4] made it necessary to change the previously assumed values of K and V , although on the whole they did not conflict with the above-mentioned simple interpretation of these quantities.

As a further check of the physical significance of the optical model parameters we carried out experiments in the summer of 1950 and spring of 1951 with the phasotron (synchrocyclotron) of the Institute for Nuclear Problems of the Academy of Sciences of the USSR, for the purpose of measuring the cross sections of inelastic interactions between 120 and 380 Mev neutrons and light and heavy nuclei (σ_a).

In order to determine σ_a we measured the neutron absorption with a threshold detector under conditions of so-called "poor" geometry [5]. Under these conditions the attenuation of the neutron beam follows the law e^{-x/λ_a} , where λ_a is the mean free path for inelastic collisions, thus making it possible to determine σ_a (from the slope of the logarithmic absorption line).

With an insufficiently high detector threshold under such conditions it is possible to record inelastically scattered neutrons, which leads to the appearance of a "transition effect" in the neutron absorption curve. Therefore, with small absorber thicknesses it is generally possible to obtain a too low value of σ_a since the slope of the logarithmic absorption curve for small thicknesses ($1-2\lambda_a$) may still differ strongly from the correct value. We overcame this difficulty by using several types of detectors with considerably different energy thresholds (the ionization-dosimetric chamber, IDC, which is a non-threshold detector; the activation of carbon by the reaction $C^{13}(n, 2n)C^{14}$ with a threshold of 20 Mev; bismuth fission chambers with a threshold of 50-60 Mev; and a tungsten fission chamber with a threshold of ~ 100 Mev). The inelastic cross sections were determined from the common slope of all the logarithmic absorption curves beyond the transition effect range.

In the 1950 experiments we used neutrons obtained by stripping 280 Mev deuterons in a copper target 16 mm thick (14.2 g/cm^2) (maximum of spectrum about 120 Mev, half-width about 40 Mev). σ_a was measured for carbon, lead and uranium. The absorbers (in the form of parallelepipeds) were placed approximately 30 m from the synchrocyclotron target. The nonuniformity of irradiation of the absorbers by the outgoing neutrons did not exceed $\pm 2\%$ (Pb, U) and 5% (C). In all cases the dimensions of absorbers and detectors were such that the possible discrepancy between the correct and measured values of the inelastic cross sections due to the "poor" geometry could not be more than a few per cent.

Measurement of the absorption of outgoing neutrons was effected by taking readings from the bismuth fission chamber and the carbon detectors. Attenuation of the neutron flux is n_1/n_0 , where $n_0 = (N_D/N_M)_0$ in the absence of an absorber (control experiment); $n_1 = (N_D/N_M)_1$ with an absorber of thickness l ; N_D is the detector count, N_M is the monitor count.

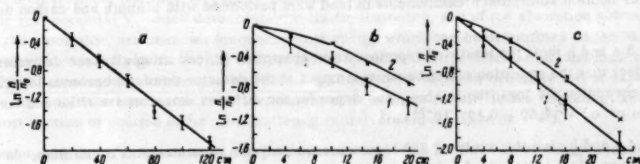


Fig. 1. Absorption of 120 MeV neutrons in carbon (a), lead (b), and uranium (c). 1) BI-detector; 2) C-detector.

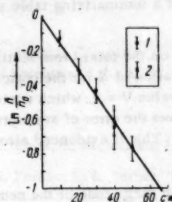


Fig. 2. Absorption of 380 MeV neutrons in aluminum. 1) BI-detector; 2) W-detector.

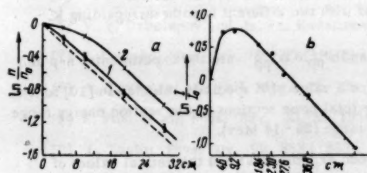


Fig. 3. Absorption of 380 MeV neutrons in lead. a) Bismuth fission chamber (1) and carbon detector (2); b) IDC.

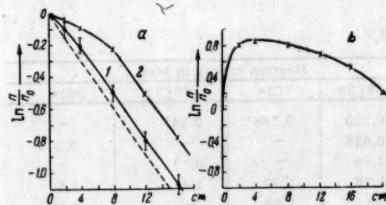


Fig. 4. Absorption of 380 MeV neutrons in uranium. a) Bismuth chamber (1) and carbon detector (2); b) IDC.

In Fig. 1, *a*, *b*, and *c* are graphs of $\ln(n_1/n_0)$ as a function of absorber thickness. The errors shown in the graphs result from the statistical scatter of the detector and monitor counts. As can be seen from *a*, *b* and *c* (curves 1) the logarithmic absorption curves of 120 MeV neutrons in carbon, lead and uranium do not reveal a "transition effect" and the inelastic cross sections given by the slopes of these graphs were: for carbon: $\sigma_a = (0.188 \pm 0.008) \cdot 10^{-24} \text{ cm}^2$; for lead: $\sigma_a = (1.50 \pm 0.16) \cdot 10^{-24} \text{ cm}^2$, and for uranium: $\sigma_a = (1.68 \pm 0.17) \cdot 10^{-24} \text{ cm}^2$.

The logarithmic neutron absorption curves in lead and uranium obtained by using carbon detectors reveal the "transition effect" at a thickness of approximately 10 cm of lead and 10-11 cm of uranium. At large thicknesses the logarithmic absorption slopes with carbon and bismuth detectors practically coincide.

In the 1951 experiments the neutrons were obtained as a result of "charge exchange" of 490 MeV protons in a 25 mm beryllium target. The neutron energy distribution in the 250-490 MeV range is given in [6]. We arbitrarily call these neutrons (from the position of the spectral maximum) "380 MeV" neutrons. In actuality, the mean effective energy when a bismuth detector is used is close to 340 MeV. In the second series of experiments we measured σ_a for aluminum, lead and uranium nuclei. The absorber dimensions and their distance from the target (6 m) were such that the nonuniformity of irradiation by neutrons did not exceed 2%.

The discrepancy between the measured and correct values of the inelastic cross-sections resulting from "poor geometry" could not exceed more than a few per cent in the majority of cases. The absorption of outgoing neutrons in aluminum was measured with a bismuth detector, and at a thickness of 40 cm with a tungsten fission chamber also. Fig. 2 shows the logarithmic absorption graph $[\ln(n_1/n_0)]$ as a function of thickness.

It is evident that the logarithmic absorption curve does not show a "transition effect" within the limits of accuracy of the measurements. From the slope for aluminum $\sigma_a = (0.258 \pm 0.023) \cdot 10^{-24} \text{ cm}^2$.

380 Mev neutron absorption measurements in lead were performed with bismuth and carbon detectors and with an IDC.

Figure 3 a and b show the results of experiments on absorption in lead using all three detectors. From this figure it is clear that the transition effect becomes stronger as the detector threshold becomes lower. Beyond the transition effect region the logarithmic absorption slopes for the different detectors are alike and correspond to $\lambda_a = 20.5 \text{ cm}$ or $\sigma_a = (1.50 \pm 0.13) \cdot 10^{-24} \text{ cm}^2$.

Figure 4 a and b give the results of 380 Mev neutron absorption measurements in uranium. In this case also the transition effect is stronger for detectors with low threshold, but past the transition effect range the different detectors give similar results. From the slope of the logarithmic curve for the bismuth detector $\sigma_a = (1.56 \pm 0.18) \cdot 10^{-24} \text{ cm}^2$.

At the conclusion of our discussion of the experimental data we present a summarizing table which includes both our results and the results of foreign authors [3, 7].

The analysis of other experiments carried out simultaneously with ours on the total cross-sections for 156 Mev neutrons [8] and 280 Mev neutrons [9], using as a basis Serber's interpretation of K for the optical model required the decrease of V to 6 Mev and to zero, respectively. This meaningless value $V = 0$, which contradicts the basic physical conception of the interaction between neutrons and nuclei, shows the error of attempting to make the theory agree with experiment on the basis of Serber's interpretation of K. This is evidenced also by an analysis of data on the inelastic cross sections of high energy neutrons.

These data make it possible to check directly various assumptions of the magnitude of the neutron absorption coefficient K independently of the magnitude of V. Our calculation, in which we used the experimental data given in Table 1, enabled us to compare the experimental values of σ_a with values obtained by different hypotheses regarding K. This calculation was carried out on the basis of the ratio

$$\frac{\sigma_a}{\pi R^2} = 1 - \frac{1 - (1 + 2KR)e^{-2KR}}{2(KR)^2} \text{ for } R = 1.37 \cdot A^{1/3} \cdot 10^{-13} \text{ cm and with two different hypotheses regarding K.}$$

Thus we assumed $K = \frac{Z\sigma'_{pr} + (A-Z)\sigma'_{pp}}{4/3\pi R^2}$ (where σ'_{pr} and $\sigma'_{pp} = \sigma_{pp}$ are the experimental n-p and p-p scattering cross-sections corrected for the Pauli principle), or a value of K specially selected in [10] for the purpose of making theory agree with experimental values of the total cross sections in the neutron energy range $E_n = 50 - 400 \text{ Mev}$, for the smallest possible change of V in this range (30 - 14 Mev).

Under both assumptions regarding K the discrepancy between experimental and theoretical values of $\sigma_a / \pi R^2$ amounts to 30-40%, especially in the higher energy region (270-400 Mev). Thus our results are evidence of the error in the choice of the parameters of the optical model, which was made only on the basis of data for the total cross sections in [10], and of the inapplicability of Serber's simple interpretation of these parameters.

TABLE 1

Inelastic Cross Section for High Energy Neutrons $\sigma_a \cdot 10^{24} \text{ cm}^2$.

Nucleus	$R \cdot 10^{13} \text{ cm}^3$ ($1.37 A^{1/3}$)	$\pi R^2 \cdot 10^{-24} \text{ cm}^2$	Neutron energy in Mev			
			90 [3]	120 •	270 [7]	380 •
C	3.35	0.352	0.223	0.188	0.143	—
Al	4.12	0.533	0.418	—	—	0.258
Cu	5.48	0.943	0.78	—	0.572	—
Pb	8.11	2.06	1.78	1.50	1.45	1.50
U	8.5	2.27	—	1.68	—	1.56

• Our results.

Agreement of the experimental data both for the total cross sections and for the inelastic cross sections in the 90-270 Mev energy range with definite values of the parameters KR and V was obtained in [11] by choosing these two parameters for a given dependence of R on A .

In the optical model one must in general consider a complex nuclear potential $U = V + iV'$ where the values of the real potential V , which determines the elastic scattering, and of the absorption potential V' , which determines the absorption, are unknown functions of the energy which can be determined on the basis of experimental results. A third parameter, the nuclear radius R , enters into the calculation, and for a complete and unique analysis it is necessary to have, in addition to the total cross sections, two more sets of experimental data, for example, the inelastic interaction cross sections (or cross sections for diffractive scattering) and the differential cross section of neutron diffractive scattering in the forward direction ($\theta = 0^\circ$).

At the present time for some nuclei there are already available all three kinds of experimental data (including data on the angular distribution of the diffracted particles) which are required for the purpose of finding $V(E)$ and $V'(E)$ (independently of definite assumptions regarding R) and for finding the nuclear radii. Such an analysis of the total data concerning nuclear interactions with high energy nucleons would be of decided interest.

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THE VERTEX PART IN SCALAR ELECTRODYNAMICS IN THE LARGE MOMENTUM REGION

A. A. Logunov

(Presented by Academician N. N. Bogoliubov July 2, 1955)

In this paper we employ a method which was described previously [1] for the purpose of finding the vertex part in the large momentum region.

The vertex part $\Gamma(k, q|1)$ for the region $|1^2| \ll |k^2|, |q^2|$ can easily be calculated on the basis of the results obtained in the previous paper [1], and agrees with the results obtained in spinor electrodynamics [2].

The vertex part $\Gamma(k, q|1)$ for $|kq| \gg |k^2|, |q^2| \gg m^2$ was obtained by a direct summation of the diagrams in which all lines of virtual quanta enclose the emission point of an external quantum $|1^2|$ [3]. The results mentioned above can easily be derived without resorting to the complicated procedure of a direct summation of diagrams.

$$e\Gamma(k, q|1) = G^{-1}(k|0) \left[\frac{\delta G(k|A)}{\delta A_1} \right]_{A=0} G^{-1}(q|0). \quad (1)$$

We set up the functional equation for

$$\Pi^V(k, l|A) = \frac{\delta G}{\delta A_l}, \quad (2)$$

$$[M^2 - \hat{H}_k^2] G(k|A) = 1, \quad (3)$$

$$\hat{H}_k^2 = \left[k^2 + \int p^2 A_+(p) \frac{\delta}{\delta A_+(p)} dp + g \int A^V(p) dp + ig \int D_{\pi}^V(p) \frac{\delta}{\delta A_+(p)} dp \right], \quad (4)$$

$$D_{\pi\nu}(p) = D(p) \left[g^{\nu\mu} - g^{\nu\rho} g^{\rho\mu} \frac{p^\nu p^\mu}{p^2} \right], \quad (5)$$

whence

$$[M^2 - \hat{H}_q^2] \Pi^V(k, q, l|A) = g [\hat{H}_q^2 + \hat{H}_k^2] G(k|A). \quad (6)$$

We give $\hat{H}_q^2$ in the form

$$\hat{H}_q^2 = \hat{H}_q^0 + \hat{R}, \quad (7)$$

where $\hat{H}_q^0$ is an operator containing terms which are not higher than the second order in g and which are linear in $A_\alpha(p)$, i.e.,

$$\begin{aligned} \hat{H}_q^0 = & q^2 + \int dp [2(pq) + p^2] A_+(p) \frac{\delta}{\delta A_+(p)} + g \int dp (2q + p) A(p) + \\ & + ig \int dp [2q^2 + p^2] D_{\pi 0}(p) \frac{\delta}{\delta A_+(p)} + ig^2 \int dp g^{\mu\nu} D_{\pi\mu}(p). \end{aligned} \quad (8)$$

Hence,

$$[M^2 - \hat{H}_q^0] \Pi(k, q, l | A) = g [\hat{H}_q^0 + \hat{H}_k^0] G(k | A) + \hat{R} \frac{\delta G(k | A)}{\delta A(l)}. \quad (9)$$

We can solve this equation if we substitute in the right hand side the expression which we calculated previously for the Green's function [1], i.e.,

$$G(k | A) = i \lim_{\epsilon \rightarrow 0} \int_0^\infty \exp[-\delta \mu + i \mu d_h + i \int \lambda_0^* A_n [e^{i \mu N} - 1] dp + \\ + i \int \lambda_0^* \frac{\Gamma_0^0 D_{ab}}{N_0} [e^{i \mu N} - 1] d\mu, \quad (10)$$

$$\lambda_0^* = -ig \frac{2k^* + p^*}{2(kp) + p^2}, \quad \Gamma_0^0 = ig [2k^0 + p^0], \quad N_0 = [2(kp) + p^2].$$

Using Fok's proper time method [4], we obtain

$$i \frac{\partial \Phi^Y(v | A)}{\partial v} = (M^2 - \hat{H}_q^0) \Phi^Y(v | A),$$

$$\Phi^Y(0 | A) = Z^Y(A_0) = - \left\{ g [\hat{H}_q^0 + \hat{H}_k^0] G(k | A) + \hat{R} \frac{\delta G(k | A)}{\delta A(l)} \right\} Z_1^{-1}, \quad (11)$$

where

$$\Pi^Y(k, q, l | A) = -i \lim_{\epsilon \rightarrow 0} \int_0^\infty \Phi^Y(v, k, q, l | A) e^{-\epsilon v} dv. \quad (12)$$

We seek the solution of equation (11) in the form

$$\Phi^Y(v | A) = \exp \left[-i \int \lambda^*(p) A_n(p) dp + i v d_q \right] F^Y(v | A), \quad (13)$$

where

$$\lambda^*(p) = -ig \frac{2q^* + p^*}{2(pq) + p^2}; \quad (14)$$

$$d_q = q^2 - M^2 + ig^2 \int dp g^{ab} D_{ab}(p) - ig^2 \int dp \frac{[2q^0 + p^0] D_{ab} [2q^* + p^*]}{2(pq) + p^2}.$$

Then

$$-i \frac{\partial F^Y(v | A)}{\partial v} = \left\{ \int dp [2(pq) + p^2] A(p) \frac{\delta}{\delta A(p)} + \right. \\ \left. + ig \int dp [2q^0 + p^0] D_{ab}(p) \frac{\delta}{\delta A_a(p)} \right\} F^Y. \quad (15)$$

Solving the last equation and substituting the solution in (13) we have, after satisfying the initial conditions:

$$\Phi^Y(v | A) = \exp \left[i v d_q + i \int \lambda^* A_n dp [e^{i v N} - 1] + \right. \\ \left. + i \int \lambda^* \frac{\Gamma^0 D_{ab}}{N} [e^{i v N} - 1] dp \right] Z^Y \left(A_n e^{i v N} + \frac{\Gamma^0 D_{ab}}{N} [e^{i v N} - 1] \right), \quad (16)$$

where $N = [2(pq) + p^2]$, $\Gamma^0 = ig [2q^0 + p^0]$, $\lambda^\alpha = -\Gamma^\alpha / N$.

Since the principal term in the initial condition (11) is the expression

$$\Phi^Y(0|A) = Z^Y(A_B) = -g[k^Y + q^Y]G(k|A)Z_A^{-1}, \quad (17)$$

we obtain after mass renormalization:

$$\begin{aligned} \Pi^Y = & gZ_A^{-1}(k^Y + q^Y) \lim_{\substack{\epsilon \rightarrow 0 \\ \delta \rightarrow 0}} \int_0^\infty d\mu \int_0^\infty dv \exp \left[-\epsilon v - \delta \mu + i v (q^2 - m^2) + i \mu (k^2 - m^2) + \right. \\ & \left. + 4i g^2 \int_0^v d\beta \int_0^\infty F(\gamma) d\gamma + 4i g^2 \int_0^\mu d\beta \int_0^\infty F(\gamma) d\gamma - i g^2 \int_0^{|\hbar q|} ds \int_0^{|\hbar q|} d\omega F(\omega, s) \right], \end{aligned} \quad (18)$$

$$F(v) = \frac{3}{2} \frac{\pi^2}{v^3} k^2 \int_0^1 e^{-i v \hbar^2 \omega} \omega d\omega \int_0^{v/\omega - v} d\beta \int_0^\beta v_t(\gamma) d\gamma, \quad (19)$$

$$\begin{aligned} F(\omega, s) = & 6\pi^2(kq)(s + \omega)^{-2} \int_0^1 d\xi \xi e^{-i \hbar^2 \xi} \int_0^{s/\xi - s} d\alpha \int_0^\alpha v_t(\gamma) d\gamma + \\ & + 4i|kq|\pi^2 s \omega \left[\left(\frac{k^2}{|kq|} \right) \left(\frac{q^2}{|kq|} \right) - 1 \right] (s + \omega)^{-2} \int_0^1 d\xi \xi^2 e^{-i \hbar^2 \xi} \int_0^{s/\xi - s} d\alpha \int_0^\alpha v_t(\gamma) d\gamma, \end{aligned} \quad (20)$$

where

$$x = \frac{s + \omega}{|kq|}, \quad \eta = \frac{\omega^2 \frac{q^2}{|kq|} + s^2 \frac{k^2}{|kq|} + 2s\omega\Delta}{s + \omega}, \quad \Delta = \frac{(kq)}{|kq|}, \quad (21)$$

$$F(v) \sim -i \frac{3}{2} \frac{\pi^2}{v^3} d_f(0), \quad F(\omega, s) \sim -2i\pi^2 \frac{1}{\omega\epsilon} d_f(0), \quad |kq| \gg |k^2|, |q^2|.$$

Equation (18) contains terms with one logarithmic expression and terms with two logarithmic expressions. Since in the momentum region $|kq| \gg |k^2|$, $|q^2| \gg m^2$ the single-logarithmic terms are unimportant, (18) can be simplified, as follows:

$$\Gamma^B(k, q|l) \sim (k^\mu + q^\mu) \exp \left[-\frac{e^2}{2\pi} \ln \left| \frac{kq}{k^2 - m^2} \right| \ln \left| \frac{kq}{q^2 - m^2} \right| \right]. \quad (22)$$

We can interpret (22) as the matrix element of the scattering of a nonfree scalar particle in an external field. As can be seen directly from this expression, the probability of such scattering is very small at sufficiently large momentum of the external quantum.

In conclusion I wish to express my deep gratitude to Academician N. N. Bogolubov, who directed this work.

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ON THE THEORY OF MULTILAYER TWO-COMPONENT DIELECTRIC COATINGS

V. V. Sukhanovskii

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The theory of multilayered dielectric coatings has been developed by many authors [1-9] but it is far from complete. Only a year ago M. P. Lisitsa [11], using the recursion formulas of A. G. Vlayov [10], obtained general equations which enable us in principle to derive all the optical properties of multilayer systems. Unfortunately, however, the equations lack rigorous mathematical derivations and are too cumbersome for the solution of many practical problems.



Fig. 1.

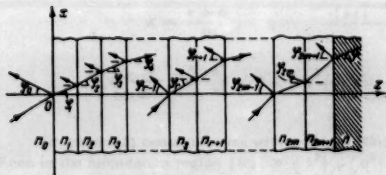


Fig. 2.

vector along the wave normal; φ_r is the angle of incidence; $\vec{\rho}$ is the radius-vector of a point with the coordinates $\{x, y, z\}$; $\mathbf{A}_r$ is the amplitude of the electric vector of the incident wave. Then for a wave reflected by the plate the following expressions hold:

$$\mathbf{E}_r^{(b)} = \mathbf{B}_r e_2; \quad \mathbf{H}_r^{(b)} = n_r [s_r^{(b)} \mathbf{B}_r] e_2. \quad (2)$$

Here $e_2 = \exp [i\omega(t - (\rho s_r^{(b)})/c_r)]$, where $s_r^{(b)} = \{\sin \varphi_r; 0; -\cos \varphi_r\}$.

Inside the plate we obviously have a system of waves which can be described by

$$\mathbf{E}_{r+1}^{(i)} = \mathbf{A}_{r+1} e_3 + \mathbf{B}_{r+1} e_4, \quad \mathbf{H}_{r+1}^{(i)} = n_{r+1} \{ [s_{r+1}^{(a)} \mathbf{A}_{r+1}] e_3 + [s_{r+1}^{(b)} \mathbf{B}_{r+1}] e_4 \}, \quad (3)$$

where $e_3 = \exp [i\omega(t - (\rho s_{r+1}^{(a)})/c_{r+1})]$; $e_4 = \exp [i\omega(t - (\rho s_{r+1}^{(b)})/c_{r+1})]$.

Resolving each of the vectors $\mathbf{A}$ and $\mathbf{B}$ into a component lying in the plane of incidence (component 1) and

In this paper we consider the case which is most frequently encountered in practice, in which a multilayer dielectric coating is formed by thin transparent and isotropic dielectric films of any two materials one of which possesses a comparatively low and the other a fairly high refractive index. Calculations on this type of coating can be carried out with the methods of matrix algebra; the present paper applies these methods to the case of all angles of incidence of a plane monochromatic wave on a two-component dielectric coating.

1. Equations of a multilayer two-component dielectric coating. On a plane-parallel transparent isotropic dielectric slab (Fig. 1) with refractive index n_{r+1} which is in a medium with the index n_r , a plane electromagnetic monochromatic wave impinges, whose electric and magnetic vectors are, respectively:

$$\mathbf{E}_r^{(a)} = \mathbf{A}_r e_1; \quad \mathbf{H}_r^{(a)} = n_r [s_r^{(a)} \mathbf{A}_r] e_1, \quad (1)$$

where $e_1 = \exp [i\omega(t - (\rho s_r^{(a)})/c_r)]$; ω is the angular velocity; c_r is the velocity of light in the medium with refractive index n_r ; $s_r^{(a)} = \{\sin \varphi_r; 0; \cos \varphi_r\}$ is a unit

vector along the wave normal; φ_r is the angle of incidence; $\vec{\rho}$ is the radius-vector of a point with the coordinates $\{x, y, z\}$; $\mathbf{A}_r$ is the amplitude of the electric vector of the incident wave. Then for a wave reflected by the plate the following expressions hold:

$$\mathbf{E}_r^{(b)} = \mathbf{B}_r e_2; \quad \mathbf{H}_r^{(b)} = n_r [s_r^{(b)} \mathbf{B}_r] e_2. \quad (2)$$

Here $e_2 = \exp [i\omega(t - (\rho s_r^{(b)})/c_r)]$, where $s_r^{(b)} = \{\sin \varphi_r; 0; -\cos \varphi_r\}$.

Inside the plate we obviously have a system of waves which can be described by

$$\mathbf{E}_{r+1}^{(i)} = \mathbf{A}_{r+1} e_3 + \mathbf{B}_{r+1} e_4, \quad \mathbf{H}_{r+1}^{(i)} = n_{r+1} \{ [s_{r+1}^{(a)} \mathbf{A}_{r+1}] e_3 + [s_{r+1}^{(b)} \mathbf{B}_{r+1}] e_4 \}, \quad (3)$$

where $e_3 = \exp [i\omega(t - (\rho s_{r+1}^{(a)})/c_{r+1})]$; $e_4 = \exp [i\omega(t - (\rho s_{r+1}^{(b)})/c_{r+1})]$.

Resolving each of the vectors $\mathbf{A}$ and $\mathbf{B}$ into a component lying in the plane of incidence (component 1) and

a component perpendicular to this plane (component 2), we find with the aid of (1), (2) and (3) that from the continuity of the tangential components of the vectors $\mathbf{E}$ and $\mathbf{H}$ at the interface $z = z_r$ it follows that

$$\psi_{r+1}^{(j)} = D_r^{(j)} \psi_r^{(j)} \quad (j = 1, 2), \quad (4)$$

where

$$\psi_{r+1}^{(j)} = \begin{pmatrix} A_{r+1}^{(j)} e^{-i\alpha'_{r+1}} \\ B_{r+1}^{(j)} e^{i\alpha'_{r+1}} \end{pmatrix},$$

$$\psi_r^{(j)} = \begin{pmatrix} A_r^{(j)} e^{-i\alpha_r} \\ B_r^{(j)} e^{i\alpha_r} \end{pmatrix}$$

is a one-column matrix and

$$D_r^{(j)} = \begin{pmatrix} d_r^{(j-1)} & d_r^{(j)} \\ d_r^{(j)} & d_r^{(j-1)} \end{pmatrix}$$

is a two-row matrix, where

$$d_r^{(j)} = 0.5 \left(\frac{\cos \varphi_r}{\cos \varphi_{r+1}} - (-1)^j \frac{n_r}{n_{r+1}} \right);$$

$$d_r^{(j+1)} = 0.5 \left(1 - (-1)^j \frac{n_r \cos \varphi_r}{n_{r+1} \cos \varphi_{r+1}} \right);$$

$$\alpha_r = \frac{2\pi}{\lambda} n_r z_r \cos \varphi_r;$$

$\alpha'_{r+1} = \frac{2\pi}{\lambda} n_{r+1} z_r \cos \varphi_{r+1}$; and λ is the wave length in vacuum.

For waves which reach the second interface $z = z_{r+1}$, we obtain by using (4):

$$\psi_{r+1}^{(j)} = M_{r+1} D_r^{(j)} \psi_r^{(j)} \quad (j = 1, 2), \quad (5)$$

where $M_{r+1} = \begin{pmatrix} e^{-i\beta_{r+1}} & 0 \\ 0 & e^{i\beta_{r+1}} \end{pmatrix}$, with $\beta_{r+1} = \frac{2\pi}{\lambda} n_{r+1} d_{r+1} \cos \varphi_{r+1}$ and $d_{r+1} = z_{r+1} - z_r$

Let us consider a set of $2m+1$ thin transparent isotropic layers of dielectric materials (Fig. 2) with refractive indices $n_1, n_2, \dots, n_r, \dots, n_{2m+1}$ placed perpendicular to the z -axis. On this set, from a medium with refractive index n_0 , let a plane monochromatic wave be incident, whose propagation vector $\mathbf{s}_0^{(j)}$ forms the angle φ_0 with the z -axis. Then, extending (4) and (5) over the entire set, we obtain

$$\psi_{2m+2}^{(j)} = D_{2m+1}^{(j)} M_{2m+1} D_{2m}^{(j)} M_{2m} \dots D_1^{(j)} M_1 D_0^{(j)} \psi_0^{(j)} \quad (j = 1, 2). \quad (6)$$

Imposing the conditions for two-component coatings, i.e. $n_{2k+1} = n_1$; $n_{2k} = n_2$; $d_{2k+1} = d_1$; $d_{2k} = d_2$ ($k = 1, 2, \dots, m$), we find that from (6) there follows

$$\psi_{2m+2}^{(j)} = D_{2m+1}^{(j)} L_j^m M_1 D_0^{(j)} \psi_0^{(j)}, \quad (7)$$

where $L_j = M_1 D_1^{(j)} M_2 D_2^{(j)}$ are matrices which characterize the coating. It can be shown that

$$L_j = \begin{pmatrix} z_{j1} - i u_{j1} & z_{j2} + i u_{j2} \\ z_{j2} - i u_{j2} & z_{j1} + i u_{j1} \end{pmatrix}$$

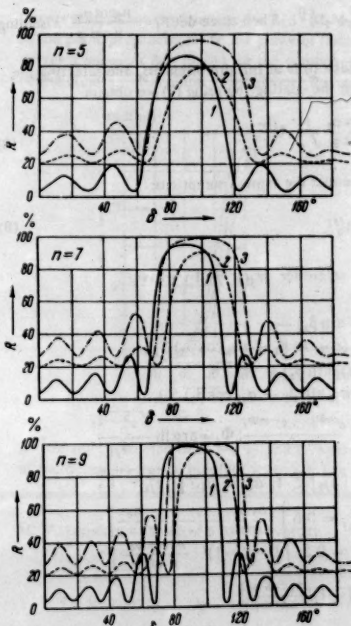


Fig. 3.

where $z_{11} = \cos \beta_1 \cos \beta_2 - 0.5 (b_1 + b_1^{-1}) \sin \beta_1 \sin \beta_2$; $u_{11} = \sin \beta_1 \cos \beta_2 + 0.5 (b_1 + b_1^{-1}) \cos \beta_1 \sin \beta_2$;
 $z_{12} = 0.5 (b_1 - b_1^{-1}) \sin \beta_1 \sin \beta_2$; $u_{12} = 0.5 (b_1 - b_1^{-1}) \cos \beta_1 \sin \beta_2$, where $b_1 = \frac{n_1 \cos \varphi_1}{n_2 \cos \varphi_2}$; $b_2 = \frac{n_1 \cos \varphi_1}{n_2 \cos \varphi_2}$.

We introduce the matrix $F_j = \begin{pmatrix} f_{j1} & f_{j2} \\ f_{j3} & f_{j4} \end{pmatrix} = D_{2m+1}^{(j)} L_j^m M_1 D_0^{(j)}$. Then since $\det F_j = \frac{n_2 \cos \varphi_2}{n \cos \varphi}$ assuming

that the backing of the coating (the medium with refractive index n) is of infinite thickness, and selecting the initial phase $\alpha_0 = 0$, for amplitudes of the light passing through the coating, by using (7) we obtain:

$$A_{2m+2}^{(j)} e^{-i\alpha'_{2m+2}} = \frac{n_2 \cos \varphi_2}{n \cos \varphi} \frac{1}{f_{j4}} A_0^{(j)} \quad (8)$$

For the amplitudes of light reflected by the coating we obtain under the same assumptions:

$$B_0^{(j)} = \frac{-f_{j2}}{f_{j4}} A_0^{(j)}. \quad (9)$$

f_{j3} and f_{j4} are obtained by the usual method of matrix algebra, where for $|z_{11}| \neq 1$ we have:

$$\begin{aligned} f_{j3} &= d_{j3} (C_j \cos \beta_1 - S_j a_{j1} \sin \beta_1) - \\ &- i [d_{j4} (C_j \sin \beta_1 + S_j a_{j1} \cos \beta_1) + d_{j2} (a_{j2} \sin \beta_1 + a_{j3} \cos \beta_1) S_j]; \\ f_{j4} &= d_{j1} (C_j \cos \beta_1 - S_j a_{j1} \sin \beta_1) + \\ &+ i [d_{j2} (C_j \sin \beta_1 + S_j a_{j1} \cos \beta_1) + d_{j4} (a_{j2} \sin \beta_1 + a_{j3} \cos \beta_1) S_j], \end{aligned}$$

where

$$C_j = (-1)^m \frac{e^{m\Phi_j} + e^{-m\Phi_j}}{2}; \quad S_j = (-1)^{m+1} \frac{e^{m\Phi_j} - e^{-m\Phi_j}}{2}; \quad \Phi_j = \arg \frac{\sqrt{z_{j1}^2 - 1}}{-z_{j1}}$$

for $|z_{j1}| > 1$; $\Phi_j = i \arg \lg \frac{V1 - z_{j1}^2}{-z_{j1}}$ for $|z_{j1}| < 1$; $a_{j1} = u_{j1} / \sqrt{z_{j1}^2 - 1}$;

$$\begin{aligned} a_{j2} &= z_{j2} / \sqrt{z_{j1}^2 - 1}; \quad a_{j3} = u_{j2} / \sqrt{z_{j1}^2 - 1}; \quad d_{j1} = 0.5 \left[\frac{\cos \varphi_0}{\cos \varphi} - (-1)^j \frac{n_0}{n} \right]; \\ d_{j1(+2)} &= 0.5 \left[\frac{n_0 \cos \varphi_1}{n_1 \cos \varphi} - (-1)^j \frac{n_1 \cos \varphi_0}{n \cos \varphi_1} \right]; \quad d_{j2} = 0.5 \left[1 - (-1)^j \frac{n_0 \cos \varphi_0}{n \cos \varphi} \right]; \end{aligned}$$

$$d_{j2(+2)} = 0.5 \left[\frac{n_0 \cos \varphi_0}{n_1 \cos \varphi_1} - (-1)^j \frac{n_1 \cos \varphi_1}{n \cos \varphi} \right]$$

2. Calculation of the reflection spectrum and phase shifts. From equations (8) and (9) we can obtain all the optical properties of multilayer two-component dielectric coatings.

TABLE 1

$\varphi_0, ^\circ$	$\Phi_1 - \pi$			$\Phi_2 - \pi$		
	$n = 5$	$n = 7$	$n = 9$	$n = 5$	$n = 7$	$n = 9$
0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2.5	0.000875	0.000963	0.000995	0.000873	0.000961	0.000992
5	0.003522	0.003878	0.004006	0.003491	0.003841	0.003965
10	0.014168	0.015631	0.016161	0.013689	0.015004	0.015508
20	0.058744	0.065317	0.067802	0.050847	0.055583	0.057207
30	0.141178	0.159649	0.167352	0.101010	0.107574	0.112693
40	0.378856	0.453435	0.495882	0.169799	0.183567	0.188161
60	0.900577	1.174254	1.418881	0.188161	0.203510	0.208952
75	2.134290	2.556555	2.981557	0.120980	0.138250	0.142687
85	2.876482	3.010361	3.136599	0.046349	0.050634	0.052398

As an example we give (see Fig. 3) the result of calculating the reflectances of 5-, 7- and 9-layered ZnS-cryolite coatings on a glass backing ($n_0 = 1$; $n_1 = 2.3$; $n_2 = 1.35$; $n = 1.5$). On the horizontal axis

$\delta = 2\pi \frac{n_1 d_1}{\lambda} = 2\pi \frac{n_2 d_2}{\lambda}$ is given in degrees; on the vertical axis the reflectance R is given in percentages.

Curves 1 represent the function $R = R(\delta)$ for $\varphi_0 = 0^\circ$; curves 2 represent $R_1 + R_2(\delta)$ for the component polarized in the plane perpendicular to the plane of incidence (electric vector in the plane of incidence) with $\varphi_0 = 45^\circ$; curves 3 represent $R_2 = R_2(\delta)$ for the component polarized in the plane of incidence (electric vector in the plane perpendicular to the plane of incidence) for $\varphi_0 = 45^\circ$.

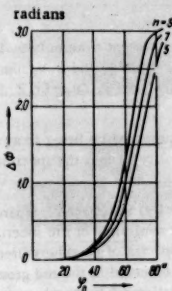


Fig. 4.

$A^{(2)}$ of light reflected from 5-, 7- and 9-layered ZnS-cryolite coatings for $\delta = 90^\circ$, as a function of the angle of incidence φ_0 from a medium with $n_0 = 1$ is given in Fig. 4.

For the solution of many practical problems it is necessary to know as precisely as possible the phase shift Φ_j of the electric vector of the reflected wave compared with the same vector in the incident wave. We can use (9) to carry out this calculation, with

$$\Phi_j = \arg \operatorname{tg} \frac{-f_{j4}}{f_{j4}} \quad (j = 1, 2). \quad \text{The results of the}$$

calculations of Φ_1 and Φ_2 for these coatings for $\delta = 90^\circ$ and various incident angles φ_0 are given in Table 1 ($n = 2m + 1$ is the number of layers of the coating).

The nature of the polarization of light reflected by a coating is determined by the phase difference $\Delta\Phi$ of the components, one of which $A^{(1)}$ is polarized in the plane perpendicular to the plane of incidence while the other $A^{(2)}$ is polarized in the plane of incidence. The phase difference $\Delta\Phi$ of the components $A^{(1)}$ and

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SOME OPTICAL PROPERTIES OF NEW RARE EARTH-ACTIVATED ZINC SULFIDE PHOSPHORS

Z. A. Trapeznikova and V. V. Shchaenko

(Presented by Academician G. S. Landsberg June 11, 1955)

Rare earth-activated zinc sulfide phosphors have been prepared up to the present time only by introducing fluxes and coactivators [1]. But by firing in various gaseous atmospheres (H_2S , NH_3 , N_2) and in vacuum we have succeeded in preparing the following phosphors: $ZnS-Nd$, $ZnS-Pr$, $ZnS-Sm$, $ZnS-Dy$, $ZnS-Er$, $ZnS-Tu$ and $ZnS-Eu$.

Figure 1 shows the luminescence spectra of these phosphors, which were prepared by firing in an ammonia atmosphere and excited by the wave length 313 $m\mu$. The $ZnS-Er$ spectrum indicates that the specimen of erbium was contaminated with thulium. *

Figure 2 shows the luminescence spectra of $ZnS-Nd$, $ZnS-Tu$ and $ZnS-Dy$, which were prepared in three different gaseous atmospheres and in vacuum. The photograph shows the great similarity of the spectra of phosphors fired in hydrogen sulfide and in ammonia and their difference from the spectra of phosphors fired in nitrogen and in vacuum. Even in the case of $ZnS-Nd$, for which such similarity is not observed in the red group of lines, the presence of a group of lines in the orange region of the spectrum sharply distinguishes phosphors prepared in hydrogen sulfide and ammonia from those prepared in nitrogen and in vacuum.

These data lead us to assume that at a high firing temperature hydrogen sulfide and ammonia show approximately the same influence on the formation of luminescence centers in phosphors, although in the first case hydrogen and sulfur may be active, and in the second only hydrogen.

Investigation of the diffuse reflection spectra of the phosphors indicated that the introduction of a rare earth ion does not lead to the appreciable formation of additional absorption bands. $ZnS-Eu$ is an exception in whose diffuse reflection curve absorption is observed in the long wave region of the spectrum; this absorption is induced by the activator. The presence in $ZnS-Eu$ of a broad emission and absorption band in the long wave region leads us to assume that Eu enters into ZnS in a bivalent state, just as it does, for example, in the lattice of SrS [2].

The edge of the fundamental absorption band is shifted towards the longer wave region by $\sim 10m\mu$ in the case of phosphors fired in vacuum and in nitrogen, which may be partly caused by the presence of a fairly large quantity of ZnO , which is always found in raw ZnS [3].

In the diffuse reflection curves very small minima are regularly observed; these correspond to the increase of absorption in the region of 360 and 380 $m\mu$. They appear especially clearly in unactivated ZnS .

In the investigation of the excitation spectra with a SF-4 spectrophotometer and a FEU-19 photomultiplier a basis of comparison was provided by luminescence from a luminogen of constant quantum efficiency and almost total absorption in the region $\lambda = 450 m\mu$ and shorter. Upon excitation of the phosphors with light from 250 to 320 $m\mu$ luminescence of the rare earth ion is observed. The excitation spectra indicate that the luminescence intensity is independent of the wave length of the exciting light, from which constancy of the yield follows, since in the measurements the exciting light was completely absorbed.

With a change of wave length from 320 to 340 $m\mu$ there is a rise of luminescence intensity with an excitation maximum at $\sim 333 m\mu$ in the phosphors $ZnS-Nd$ (see Fig. 3, a), $ZnS-Pr$, $ZnS-Sm$, $ZnS-Dy$ and $ZnS-Er$ **, and with an excitation maximum at $\sim 328 m\mu$ in $ZnS-Tu$ (Fig. 3, b). The luminescence spectrum

- * A part of the thulium spectrum in the blue region of Fig. 1 was omitted in order to save space.
- ** The excitation curves of these phosphors are not given, as they are similar to the excitation curve of $ZnS-Nd$.

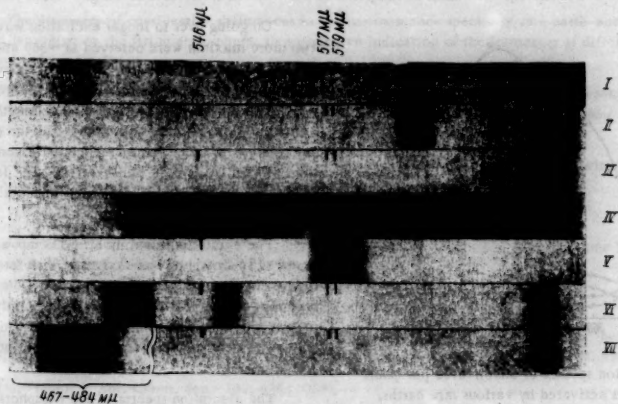


Fig. 1. Photographs of the luminescence spectra of phosphors excited by 313 mμ. I) ZnS-Pr, II) ZnS-Nd, III) ZnS-Sm, IV) ZnS-Eu, V) ZnS-Dy, VI) ZnS-Er, VII) ZnS-Tu.

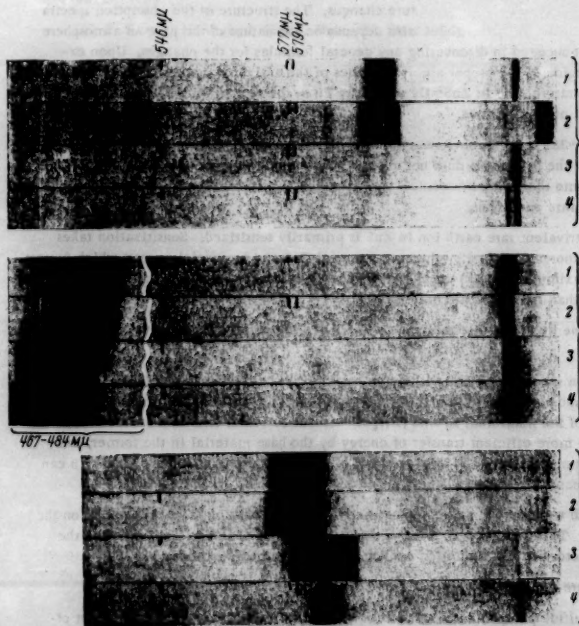


Fig. 2. Photographs of the luminescence spectra of the phosphors: ZnS-Nd (I), ZnS-Tu (II), and ZnS-Dy (III), fired in different gaseous atmospheres: 1) in ammonia; 2) in hydrogen sulfide; 3) in nitrogen; 4) in vacuum.

corresponding to the excitation maximum is composed of the characteristic lines of the given rare earth ion.

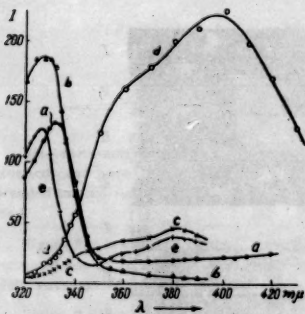


Fig. 3. Excitation spectra of unactivated phosphors and of phosphors activated by various rare earths. a) ZnS-Nd; b) ZnS-Tu, fired in a hydrogen sulfide atmosphere; c) unactivated ZnS fired in vacuum; d) ZnS-Eu fired in hydrogen sulfide; e) unactivated ZnS (NH_4Cl), fired in vacuum.

during firing. However, we did not succeed in discovering any general formulas for the changes. Upon excitation with visible light in the region of the proper absorption lines of the activator weak luminescence was observed in ZnS-Er at room temperature, and in ZnS-Dy and ZnS-Tu at the temperature of liquid nitrogen. These luminescence spectra consist of narrow lines.

An absorption maximum at $\sim 333 \text{ m}\mu$ can obviously not be explained as being due to absorption by the rare earth ion, since the position of the maximum does not change with the introduction of different rare earth ions. One might more readily assume that there are centers in ZnS which can absorb light in this region and transfer the absorbed energy to the rare earth ions.

Thus the luminescence of a trivalent rare earth ion in ZnS is primarily sensitized. Sensitization takes place as a result of the transfer of energy absorbed by the base material ZnS and by special centers which possess a narrow absorption band with a maximum at $\sim 333 \text{ m}\mu$. The efficiency of the transfer of energy from these centers is, as a rule, much greater than from the base material.

It is most reasonable to assume that these are the centers which are responsible for the blue luminescence of unactivated ZnS.

Investigation of the excitation spectrum of ZnS without an activator and with the flux NH_4Cl , showing blue luminescence, also indicated the existence of an excitation maximum at $\lambda = 328 \text{ m}\mu$. It can be assumed that the difference in the position of the maximum in ZnS (NH_4Cl) without activator and ZnS activated with a rare earth is to be explained by the more efficient transfer of energy by the base material in the former, thus resulting in a shift of the excitation maximum. The displacement of the excitation maximum of ZnS-Tu can possibly be explained by the same cause or by the existence of proper absorption by Tu in this region.

The excitation maxima which we observed agree approximately with the data of K. Shalimova [4] on the absorption of sublimated phosphors. This indicates, evidently, that both kinds of phosphors are formed in the same way. But the chemical character of their formation cannot be considered fully explained [4, 5].

Thus the results of our work are the following:

1. The preparation of zinc sulfide phosphors activated with rare earths and without the employment of

On going over to longer excitation wave lengths two more maxima were observed at ~ 360 and $\sim 380 \text{ m}\mu$. In Fig. 3, a and b, these maxima are not given as they do not represent luminescence caused by rare earth ions. As already stated above, at these wave lengths a small amount of absorption is also observed in the diffuse reflection curves. The luminescence corresponding to these maximum is light bluish-green. The last two excitation maxima are also found in unactivated ZnS (Fig. 3, c).

The excitation maxima at $\sim 333 \text{ m}\mu$ are as a rule observed in ZnS phosphors activated with rare earths, independently of the firing atmosphere. The excitation spectrum of ZnS-Eu differs sharply; here only an irregular broad absorption band is observed which probably consists of two bands corresponding to absorption in the activator (Fig. 3, d).

The absorption spectra of the phosphors ZnS-Nd, ZnS-Pr, ZnS-Dy and ZnS-Er show a clear line structure in the visible region. The position of the absorption lines and their relative intensities shows strong temperature changes. The structure of the absorption spectra also depends on the nature of the gaseous atmosphere

fluxes or coactivators.

2. The discovery of considerable differences in the luminescence spectra of rare earth-activated ZnS phosphors depending on the firing atmosphere, thus giving an indication of the formation of different luminescence centers under these conditions.

3. It was established that luminescence of a trivalent rare earth ion in these phosphors results from the transfer of energy absorbed by the base material and by special centers which absorb in a narrow spectral band around 333 m μ .

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ELECTRON ENERGY LEVELS AND INTERATOMIC DISTANCES IN MOLECULES*

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In connection with the complicated problem of the relationship between electron energy levels and other molecular properties we have formulated several theorems which are interesting even if only because of their simplicity.

I. Diatomic molecules. A comparison is drawn between diatomic molecules consisting of atoms of the same group of the periodic table**, especially if the atoms are identical (for example, Cl_2 , Br_2) or if one of them is repeated (NaH , KH). From experimental data*** (see tables) it is evident that for such molecules:

I. There is a relationship between the values T_1 of the optical electron terms and the equilibrium inter-nuclear distance l .

II. For molecules with greater length of l the terms are lower and the wave lengths λ of transitions are longer (the same connection between λ and l has also been observed in connection with several other matters [3]).

III. The following rule holds: electron terms are approximately inversely proportional to the bond lengths l : $T_1 l = \text{const}$. Hence for transitions of the same type $\nu l = \text{const}$, and the wave lengths are proportional to the length of the molecule (analogy: string - wave length proportional to length of string).

Table 1 refers to the terms X and A which determine the spectral color; for a number of molecules of the table the transitions $A \longleftrightarrow X (\nu = T_1)$ lie in or close to the visible region.

From the table it can be seen that electron configurations and terms of homologous molecules are of the same type and that rule III is observed: the product $T_1 l$ is approximately constant, changing very much less than T_1 and l taken separately. For example, for molecules of the first group, the relative change Δ in per cent is: for l 146%, for T_1 127%, for $T_1 l$ only 9%. In view of the approximate nature of the rule the appreciable inconsistency of the data for NaH is not significant. l for NaK is not found in [1, 2]; we have taken the average of the lengths for Na_2 and K_2 .

From Tables 2 and 3 it can be seen that $T_2 l = \text{const}$ and $l = \text{const}$. Empirical data can also be used to confirm the rule for intermediate terms. For this purpose it is only essential to compare terms (ionization schemes) of the same type (see Tables 1 and 2; in Table 3 for all hydrogen halides, for example, there is only one ionization scheme: $\text{H-Hal} (^2\Sigma) \longrightarrow \text{H-Hal} (^2\Pi) + e$, where $\text{Hal} = \text{Cl}, \text{Br}, \text{I}$). On the other hand, if the rule is observed we may assume that the terms (or ionization schemes) of the homologous molecules being

* Delivered upon assuming the Chair of Theoretical Physics and Astronomy of the Herzen Pedagogical Institute, November 9, 1954.

** For the sake of brevity we shall call them homologous molecules.

*** Data and notation unless otherwise indicated are from [1, 2]. Terms in order of increasing magnitude are denoted by letters: $X, A, B, \dots$; their magnitudes in cm^{-1} are correspondingly: $T_0, T_1, T_2, \dots$; T_0 is taken to be zero, so that the T_k coincide with the wave numbers ν_k of transitions where the lower level is the ground level (in particular, absorption spectra). The ionization potential J is in volts. Bond lengths l in $\text{\AA}$ are taken for the ground state.

compared are of the same type, although it is not immediately apparent if the symbols ascribed to these terms are correct, since the rule evidently refers to any term.

TABLE 1

Molecule	Zero term		First term		T ₁	T ₁ ¹		
	Symbol	Bond length <u>l</u>	Symbol	T ₁				
Molecules of the first group								
LiH	2sσ ²	X ¹ Σ ⁺	1.50	2sσ2pσ	A ¹ Σ ⁺	26516	42100	67000
LiD	2sσ ²	X ¹ Σ ⁺	1.59	2sσ2pσ	A ¹ Σ ⁺	26512	42100	67000
NaH	3sσ ²	X ¹ Σ ⁺	1.88	3sσ3pσ	A ¹ Σ ⁺	23696	44500	83500
			1.89	3sσ3pσ	A ¹ Σ ⁺	22719	42900	81000
KH	4sσ ²	X ¹ Σ ⁺	2.24	4sσ4pσ	A ¹ Σ ⁺	19530	43600	97500
KD	4sσ ²	X ¹ Σ ⁺	2.30	4sσ4pσ	A ¹ Σ ⁺	19585	45100	104000
RbH	5sσ ²	X ¹ Σ ⁺	2.37	5sσ5pσ	A ¹ Σ ⁺	18906	44600	106000
CsH	6sσ ²	X ¹ Σ ⁺	2.49	6sσ6pσ	A ¹ Σ ⁺	18409	45800	114000
Na ₂		X ¹ Σ _g ⁺	3.07		A ¹ Σ _g ⁺	14680	45000	138000
NaK		X ¹ Σ ⁺	3.40		A ¹ Σ ⁺	12140	42300	147500
K ₂		X ¹ Σ _g ⁺	3.91		A ¹ Σ _g ⁺	11684	45500	178000
Δ			146%			127%	9%	166%
Ions of hydrides and deuterides of the zinc group								
ZnH ⁺	4sσ ²	X ¹ Σ ⁺	1.51	4sσ4pσ	A ¹ Σ ⁺	46700	70500	106500
CdH ⁺	5sσ ²	X ¹ Σ ⁺	1.66	5sσ5pσ	A ¹ Σ ⁺	42934	71000	118000
HgH ⁺	6sσ ²	X ¹ Σ ⁺	1.59	6sσ6pσ	A ¹ Σ ⁺	44319	70500	112000
HgD ⁺	6sσ ²	X ¹ Σ ⁺	1.59	6sσ6pσ	A ¹ Σ ⁺	44307	70500	112000
Δ			10%			9%	0.7%	11%
Boron halides								
¹⁰ BF		X ¹ Σ ⁺	1.26		A ¹ Π	51151	64500	81200
H ¹⁰ B ³⁵ Cl		X ¹ Σ ⁺	1.72		A ¹ Π	36750	63000	108500
H ¹⁰ B ⁷⁹ Br		X ¹ Σ ⁺	1.89		A ¹ Π	33953	64100	121000
Δ			50%			51%	2.5%	40%
Aluminum halides								
Al ²³ Cl		X ¹ Σ _g ⁺	2.14		A ¹ Π ₁	38254	81600	175000
Al ²⁷ Br		X ¹ Σ _g ⁺	2.29		A ¹ Π ₁	35879	82000	188000
Δ			7%			6.5%	0.5%	7.5%
Halogens								
Cl ₂		X ¹ Σ _g ⁺	1.99		³ Π ₀₋₂	18310	36400	72500
Br ₂		X ¹ Σ _g ⁺	2.28		³ Π ₀₋₂	15891	36200	82500
BrF		X ¹ Σ ⁺	1.76		³ Π ₀₋₂	19335	34000	59600
Δ			29%			22%	7%	38%

We shall now touch on the theoretical interpretation of rule III. The mutual potential energy U of a valence electron and the atomic remainders in a diatomic molecule is (subject to certain limitations)

$$U = \frac{c_0}{1} - \frac{c_1}{r_1} - \frac{c_2}{r_2}, \text{ where } r_1 \text{ and } r_2 \text{ are the distances of the electron from the atomic cores; the constants } c_0,$$

c_1 and c_2 also take into account the degree of screening of the nuclear charges by the electrons of the atomic cores*. If we assume that for a homologous molecule (indicated by primes) there is observed a similarly

* The model of a potential well with infinitely high walls which is used in other cases is unacceptable here; for homologous molecules it would lead to constancy of T_1^1 , which is contrary to experiment— T_1^1 changes more than T_1 (Table 1).

in the configuration, in particular, of the valence electron, i.e., $\frac{r_1^2}{r_1} = \frac{r_2^2}{r_2} = \frac{l^2}{1} = \epsilon = \text{const}$ and $c_0^2 = c_0$, $c_1^2 = c_1$, $c_2^2 = c_2$, where

$$U' = \frac{c_0}{l^2} - \frac{c_1}{r_1} - \frac{c_2}{r_2} = \frac{c_0}{\epsilon l} - \frac{c_1}{\epsilon r_1} - \frac{c_2}{\epsilon r_2} = \frac{U}{\epsilon} = \frac{l}{l'} U,$$

the time average $\bar{U} = \frac{1}{l'} \bar{U}$; $\frac{\bar{U}^*}{\bar{U}} = \frac{1}{l'}$. $\bar{U}$ changes inversely with l . According to the virial theorem (which also holds in quantum mechanics) the energy levels (normalized at infinity) will also be inversely proportional to l :

$$E_h' = \frac{E_h}{\epsilon}; \quad \frac{E_h'}{E_h} = \frac{l}{l'}.$$

TABLE 2

Molecule	Zero term		Second term		T ₂ ¹
	symbol	bond length	symbol	T ₂	
		$\frac{l}{1}$			
Alkali molecules					
Na ₂	X ¹ Σ _g ⁺	3.07	B ¹ Π _u	20319	62400
K ₂	X ¹ Σ _g ⁺	3.91	B ¹ Π _u	15378	60300
NaK	X ¹ Σ ⁺	3.49	B ¹ Π	16991	59300
Δ		27%		32%	5%
Oxygen group					
O ₂	X ³ Σ _g ⁻	1.21	B ³ Σ _u ⁻	49802	60200
S ₂	X ³ Σ _g ⁻	1.89	B ³ Σ _u ⁻	31835	60100
Se ₂	(X)	2.15	B	26035	56000
Te ₂	X	2.59	B	22714	58800
SO	(X ³)Σ ⁻	1.49	B ³ Σ ⁻	39356	58600
Δ		114%		120%	7.5%

TABLE 3

Molecule	Bond length $\frac{l}{1}$	Ionization potential J	Π
Hydrogen halides			
HCl	1.27	12.84	16.3
HBr	1.41	12.04	17.0
HI	1.60	10.71	17.1
Δ	26%	20%	5%
CO and CS			
CO	1.125	14.1	15.85
CS	1.53	10.6	16.2
Δ	36%	33%	2%

TABLE 4

Molecule *	Bond	Bond length $\frac{l}{1}$	$\nu_{\min}$ (for vapor)	$\nu_{\min}^1$	$\nu_{\min}^{1*}$
CH ₃ Cl, C ₂ H ₅ Cl	C-Cl	1.77	62000	109500	193500
CH ₃ Br, C ₂ H ₅ Br	C-Br	1.91	55600	106000	202000
CH ₃ I, C ₂ H ₅ I, (CH ₃) ₂ CHI					
C ₆ H ₅ I, (CH ₃) ₂ CHI	C-I	2.28	47500	108300	247000
Δ		28.5%	30.5%	3.3%	27.5%
C ₆ H ₅ -O-C ₆ H ₅	C-O	1.43	53200	76000	108400
C ₆ H ₅ -S-C ₆ H ₅ , S(CH ₃) ₂ , CH ₃ -S-C ₆ H ₅	C-S	1.81	43700	79000	143000
Δ		26.5%	22%	4%	32%

* Since the experimental values of l and $\nu_{\min}$ (i. e., ν for the "red" absorption edge) agree approximately for the molecules of Table 4 with the same halogen, only the first is given here. These ν , because of several considerations, can reasonably be ascribed to excitation of the carbon-halogen "bond". A similar statement may be made with regard to the others.

* This also refers to the usual case of homopolar binding involving two electrons if it is assumed that for their mutual separation $r_{12}/r_{12} = 1/l$. Ionic molecules (salts) are not considered in the present short paper.

The terms

$$T_h = \frac{E_h - E_0}{hc}; \quad \frac{T'_h}{T_h} = \frac{E'_h - E'_0}{E_h - E_0} = \frac{E_h - E_0 - \epsilon}{E_h - E_0} = \frac{1}{\epsilon} = \frac{1}{I'}$$

or

$$T'_{k1} = T_{k1} = \text{const}$$

are the same for the different homologous molecules. We have now obtained rule III. Of importance here is:
1) the potential energy is a homogeneous function of degree -1 of the mutual separations of the particles;
2) (average) similarity.

The greater the similarity the better the rule is observed. For example, in Na and also in K the valence electron is adjacent to a shell with 8 electrons, but in Li there are only two other electrons, and similarity with Li is not so well observed. For Na₂ and K₂, T₁ are very close: 45,000 and 45,500; for Li₂ we have, on the other hand, 37,500. For secondary subgroups, due to the peculiarities of the electron shells which manifest themselves, in particular, in variable valence, rule III is generally not observed. Thus, for CuH, T₁ is equal to 34,200; for AgH it is 48,000; for AuH it is 41,800 - a change of 40%. In the Zn group, which is also secondary, but where the valence is constant, the rule is somewhat better observed for hydrides (T₁ changes by ~15%), which is evidently associated with the comparative smallness of their dissociation energies. This may perhaps be interpreted as follows: the rule is always observed if we consider the change of terms due only to a transformation where similarity is preserved, but this change is in some cases masked by the effect of secondary factors.

TABLE 5

Mole- cule	Bond	Bond length, Å	J ₁	J ₁ '	J ₂	J ₂ '	J ₃	J ₃ '	J ₄	J ₄ '	J ₅	J ₅ '	J ₆	J ₆ '
CO	O=C	1.15	CO ₂ ⁻ -CO ₂ ⁻ (H) ₂ + 18.5 ±0.01	15.8	CO ₂ ⁻ -CO ₂ ⁻ O ⁺ + 19.6 ±0.4	22.6	CO ₂ ⁻ -CO ₂ ⁻ O ⁺ + 20.4 ±0.7	23.5	CO ₂ ⁻ -C ⁺ +2O ⁺ + 28.3 ±1.5	32.6	CO ₂ ⁻ -CO ₂ ⁻ +2e ⁻ 82 ±2	56.8	C ⁺ +2O ⁺ +2e ⁻ 55 ±3	69.2
CS	S=C	1.58	CS ₂ ⁻ -CS ₂ ⁻ (H) ₂ + 10.0	15.7	CS ₂ ⁻ -CS ₂ ⁻ S ⁺ + 14.0 ±0.5	21.8	CS ₂ ⁻ -CS ₂ ⁻ S ⁺ + 14.7 ±0.5	22.0	CS ₂ ⁻ -C ⁺ +2S ⁺ + 21.5 ±1	33.6	CS ₂ ⁻ -CS ₂ ⁻ +2e ⁻ [38.8]	[56.8]	C ⁺ +2S ⁺ +2e ⁻ [40.5]	[63.2]
Δ			35.5%	87%	0.5%	40%	3.7%	80%	2.0%	31.8%	8%			

2. Polyatomic molecules. Tables 4 and 5 show that rule III can be applied also to "homologous" bonds in polyatomic molecules and, as is especially interesting, to various (i.e., for correspondingly different ionization schemes) ionization potentials. The expressions in square brackets in Table 5 have been predicted on the basis of the rule: since J₁ for CS₂ almost agrees with the corresponding values for CO₂ for k = 1, 2, 3, 4, it is reasonable to expect agreement for k = 5, 6* also; hence (1 being unknown) it is possible to obtain for CS₂ the unknown values of J₅ and J₆. By employing rule III it is possible in many cases to make predictions.

* Unfortunately, the error in the values of J₅ and J₆ for CO₂ is comparatively large.

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FAST PARTICLE ABSORPTION CROSS SECTION OF SEMITRANSSPARENT NUCLEI

S. I. Drozdov

(Presented by Academician M. A. Leontovich July 15, 1955)

1. We shall examine the scattering of neutrons by absorbing nuclei. Let the interaction energy of the particle and the nuclei be of the form

$$V(r) = -V_0(r) - iV_1(r), \quad (1)$$

where r is the distance between the particle and the center of the nucleus. We assume that the functions $V_0(r)$ and $V_1(r)$ for $r \leq R$ are constant, and that for $r > R$ they fall off to zero at a distance $\Delta r \ll R$. According to the phase theory of scattering, the calculation of the effective cross sections reduces to finding the phase of the radial wave function at infinity. This function satisfies the equation

$$\left(\frac{d^2}{dr^2} + k_l^2(r)\right)u_l(r) = 0; \quad (2)$$

$$k_l^2(r) = K^2(r) - \frac{l(l+1)}{r^2}; \quad K^2(r) = k^2 + \frac{2m}{\hbar^2} V(r),$$

where $k^2 = 2mE/\hbar^2$ is the wave vector of the incident particle and l is the orbital moment.

Equation (2) will be solved approximately by the semiclassical method of M. I. Petrashen [1]. A similar approximation is given in [2]. Using the indicated method we obtain:

$$u_l(r) = \left(\frac{dx(r)}{dr}\right)^{-1/2} \sqrt{Kx(r)} J_{l+1/2}(Kx(r)), \quad (3)$$

where $K = K(r)$, $r < R$; $J_{l+1/2}(z)$ is a Bessel function of the first kind of half-integral order. The function $x(r)$ is determined from the equation

$$\int_0^{x(r)} \sqrt{(xK)^2 - l(l+1)} \frac{dx}{x} = \int_0^r \sqrt{(rK(r))^2 - l(l+1)} \frac{dr}{r}. \quad (4)$$

For $r < R$, $x(r) = r$. For $r \rightarrow \infty$

$$\int_R^{x(r)} \sqrt{(xK)^2 - l(l+1)} \frac{dx}{x} \approx \int_R^r \sqrt{(kr)^2 - l(l+1)} \frac{dr}{r}, \quad (5)$$

whence it follows that

$$Kx(r) \approx kr + b_l;$$

$$\delta_l = \sqrt{(kR)^2 - l(l+1)} - \sqrt{(kR)^2 - l(l+1)} + \\ + \sqrt{l(l+1)} \left(\arccos \frac{\sqrt{l(l+1)}}{kR} - \arccos \frac{\sqrt{l(l+1)}}{KR} \right) \quad (6)$$

Examination of the asymptotic form of (3) shows, with the aid of (5), that δ_l is the phase of the wave function at infinity.

Equation (6) is correct if the condition for the quasi-classical approximation is satisfied:

$$\left| \frac{dk_l(r)}{dr} \right| \ll |k_l^2(r)|, \quad (7)$$

which must be observed in the region $r \gg R$ where the interaction potential (1) falls off. In the derivation of (6) the centrifugal potential was included exactly. Therefore, unlike the Wentzel-Kramers-Brillouin (WKB) method, which is applicable only for $l \gg 1$, (6) can be used for all $l \leq kR$. At $l > kR$ the condition for applicability may be violated. However, in the calculation of the cross-sections this circumstance is unimportant if it is assumed that the particle interacts with the nucleus only for $l < kR$.

It is easy to verify a few relationships in the limit.

1) For an absolutely "black" nucleus when the absorption coefficient $\mu = 2 \operatorname{Im} K \rightarrow \infty$ the amplitude of the scattered wave $\eta_l = e^{2i\delta_l}$ vanishes.

2) At $V_1(r) = 0$, $l \gg 1$, the magnitude of δ_l agrees with the phase obtained by the WKB method (this method cannot be used for a complex interaction potential).

3) Equation (6) takes into consideration the refraction of the incident wave striking the nucleus. For very high energies, when the refraction coefficient $\bar{n} = \operatorname{Re} K/k$ is close to 1, (6) agrees with the results of the calculation in [3] which neglects refraction:

$$\delta_l \approx \frac{K-k}{k} \sqrt{(kR)^2 - l^2}; \quad \left| \frac{K-k}{k} \right| \ll 1; \quad l \gg 1.$$

According to the phase theory of scattering [4], the neutron absorption cross section for $kR \gg 1$ can be represented as

$$\sigma_c = \frac{\pi}{k^2} \int_0^{kR} dl (2l+1) (1 - |e^{2i\delta_l}|^2); \quad kR \gg 1. \quad (8)$$

The integral in (8) with δ_l given by (6) cannot be calculated in infinite form. It may conveniently be represented in the form of a power series in the ratio $(k/|K|)^2 < 1$. This series can easily be obtained by expanding in powers of Z the function

$$|e^{2i\delta_l}|^2 = |e^{2iK/(2)}|; \quad f(z) = \sqrt{1-z^2} - z \arccos z; \quad z = \frac{\sqrt{l(l+1)}}{K}.$$

Thus we find

$$\sigma_c = \pi R^2 \left\{ 1 - e^{-2\mu R} \sum_{n=0}^{\infty} b_n \left(\frac{k}{|K|} \right)^{2n} \right\}; \quad kR \gg 1; \quad (9)$$

$$\mu = 2 \operatorname{Im} K; \quad b_0 = 1; \quad b_1 = \frac{\mu R}{2}; \quad b_2 = \frac{1}{6} \left(\frac{\mu R}{2} + (\mu R)^2 + \frac{\mu^2 R}{6 |K|^2} \right), \dots$$

where for $k/|K| \ll 1$

$$\sigma_c = \pi R^2 (1 - e^{-2\mu R}); \quad kR \gg 1; \quad \frac{k}{|K|} \ll 1. \quad (10)$$

Equation (10) shows that the incident wave, when it impinges on the nucleus, undergoes maximum refraction and is propagated in the nucleus normal to the surface, i.e., along the diameter $2R$. From (10) and the expression for σ_c in [3] it follows that in the cases both of small and large refraction the absorption cross-section is determined by the radius of the nucleus and by its absorption coefficient. In the intermediate case (8) the expression for the absorption cross-section is more complicated.

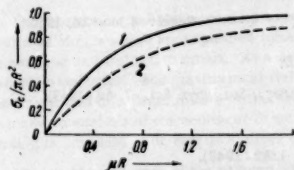


Fig. 1. Curve of the cross-section $\sigma_c(\mu R)$ including refraction (1), and neglecting refraction (2).

Figure 1 shows as functions of the absorption coefficient μR the absorption cross-section given by (10) and the cross-section σ_c obtained in [3] by neglecting refraction. As can be seen from the curve, refraction contributes considerably to the cross-section for $\mu R \lesssim 1$.

2. We shall examine in an analogous manner the scattering of charged particles. We neglect the parabolic addition to the potential of the electrical interaction of the particle with the nucleus for $\underline{r} < R$. In this case $k_1(\underline{r})$ in (2) can be represented as

$$k_1^2(\underline{r}) = \begin{cases} K^2 - \frac{2mZZ'e^2}{\hbar^2 R} - \frac{l(l+1)}{r^2}, & r \leq R; \\ K^2(r) - \frac{2mZZ'e^2}{\hbar^2 r} - \frac{l(l+1)}{r^2}, & r \geq R, \end{cases}$$

where Ze is the charge of the nucleus and $Z'e$ is the charge of the particle. Employing the method of M. I. Petrashen, we give the radial wave function as

$$u_l(r) = \left(\frac{dx(r)}{dr}\right)^{-1/2} \sqrt{K'(x(r))} J_{l+1/2}(K'(x(r))); \quad K'^2 = K^2 - \frac{2mZZ'e^2}{\hbar^2 R}, \quad (11)$$

where $x(r)$ for $\underline{r} \rightarrow \infty$ is determined from the equation

$$\int_R^{x(r)} \sqrt{K'^2 - \frac{l(l+1)}{x^2}} dx \approx \int_R^r \sqrt{k^2 - \frac{2mZZ'e^2}{r} - \frac{l(l+1)}{r^2}} dr. \quad (12)$$

In accordance with (8) for calculation of the absorption cross-section it is sufficient to know only the imaginary part of the phase δ_l . The result of calculations similar to the corresponding calculations for neutrons gives from (11) and (12)

$$\text{Im } \delta_l = \text{Im} \left(\sqrt{(K'R)^2 - l(l+1)} - \sqrt{l(l+1)} \arccos \frac{\sqrt{l(l+1)}}{K'R} \right). \quad (13)$$

If we neglect the interaction with the nucleus when the energy of the particle is below the potential barrier we obtain

$$\sigma_c = \frac{\pi}{k^2} \int_0^{l_0} d(2l+1) (1 - e^{-4 \text{Im } \delta_l}); \quad l_0 = kR \sqrt{1 - \frac{ZZ'e^2}{RE}} \gg 1.$$

As in the case of neutrons it is convenient to represent this integral as a series:

$$\sigma_c = \frac{\pi}{k^2} l_0^2 \left\{ 1 - e^{-\mu R} \sum_{n=0}^{\infty} b_n \left(\frac{l_0}{|K'|R} \right)^{2n} \right\};$$

$$l_0 \gg 1; \quad \mu = 2 \operatorname{Im} K'; \quad b_0 = 1; \quad b_1 = \frac{1}{2} \mu R, \dots$$

(14)

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NONLINEAR GENERALIZATION OF THE DIRAC SPINOR EQUATION

D. Ivanenko and M. Mirianashvili

(Presented by Academician N. N. Bogoliubov April 23, 1955)

The interaction of fields according to relativistic quantum mechanics leads in general to the appearance of nonlinear effects which are explained intuitively as being due to the possibility of the transformation of some particles and fields into others. As a continuation of papers in which the nonlinearities in scalar, pseudo-scalar, electromagnetic and gravitational fields were examined, we shall here examine spinor fields. The case of the interaction of a Dirac field χ with another given Dirac field ψ is of particular interest since it enables us to make an analysis of the nonlinear Dirac equation which describes electrons and nucleons and which may, according to the ideas of de Broglie's theory of fusion, even be the basis of the theory of other fields.

We consider the state of the vacuum with respect to the field χ , and give the Lagrangian of the interaction in the form

$$L' = -g\bar{\chi}\gamma_5\chi \cdot \bar{\psi}\gamma_5\psi. \quad (1)$$

Then the equations for the fields $\psi(x)$ and $\chi(x)$ are of the form

$$\begin{aligned} (i\gamma_\mu\partial_\mu - g\gamma_5\bar{\chi}\gamma_5\chi + m)\psi(x) &= 0, \\ (i\gamma_\mu\partial_\mu - g\gamma_5\bar{\psi}\gamma_5\psi + m)\chi(x) &= 0, \end{aligned} \quad (2)$$

where g is the binding constant with the dimension of the Fermi constant g_F in β -decay theory; the letter m in our units ($\hbar = c = 1$) obviously designates the quantity $mc/\hbar$. Assuming that $\psi(x)$ is a given field and that the vacuum state is taken with respect to the field χ , we obtain

$$(-i\gamma_\mu\partial_\mu + m)\psi(x) = g\gamma_5\psi(x)\langle\bar{\chi}\gamma_5\chi\rangle_{\text{vac}} = \langle J(x)\rangle_{\text{vac}}, \quad (3)$$

where $\langle J(x)\rangle_{\text{vac}}$ is the vacuum value of the spinor current:

$$\langle J(x)\rangle_{\text{vac}} = g\gamma_5\psi(x)\langle\bar{\chi}\gamma_5\chi\rangle_{\text{vac}} = ig\gamma_5\psi(x)\text{Sp}\gamma_5 G(x, x')|_{x'=x}, \quad (4)$$

and $G(x, x')$ is the Green's function of the field $\chi(x)$, which satisfies the equation

$$(-i\gamma_\mu\partial_\mu - g\gamma_5\bar{\psi}\gamma_5\psi + m)G(x, x') = \delta(x - x'), \quad (5)$$

or in operator form

$$(\gamma p - g\gamma_5\bar{\psi}\gamma_5\psi + m)G = 1.$$

The addition to the action function associated with the vacuum "spinor polarization current" will be

$$\delta W^{(1)} = \int (dx) \delta\bar{\psi}(x) \langle J(x)\rangle_{\text{vac}} = ig \text{Sp} \delta\bar{\psi}(x) \gamma_5 \psi(x) \gamma_5 G, \quad (6)$$

and the addition to the Lagrangian density

$$\delta L^{(1)} = ig \delta\bar{\psi}(x) \gamma_5 \psi(x) \text{Sp} \gamma_5 G. \quad (7)$$

Hence by Schwinger's method [1] and using his notation we obtain

$$L^{(1)} = \frac{1}{2} (2\pi)^{-3} \int_0^\infty s^2 \exp[-i(m^2 + g^2(\bar{\psi}\gamma_5\psi)^2)s] ds; \quad (8)$$

$$\langle j(x) \rangle_{\text{vac}} = -i g^2 \gamma_5 \psi(x) \bar{\psi}(x) \gamma_5 \psi(x) \times \\ \times (2\pi)^{-3} \int_0^\infty ds \cdot s^{-3} \exp \{-i [m^2 + g^2 (\bar{\psi} \gamma_5 \psi)^2] s\}. \quad (8)$$

Expanding the exponential function in powers of $(\bar{\psi} \gamma_5 \psi)$ and stopping at the first term we obtain

$$L^{(1)} = \frac{g^2}{8\pi^2} \left[\frac{1}{s_0} - m^2 \ln \frac{1}{\gamma m^2 s_0} - m^2 \right] (\bar{\psi}(x) \gamma_5 \psi(x))^2 \quad (9)$$

or

$$L^{(1)} = -\frac{m^2}{8\pi^2} \frac{g^2}{\hbar c} \left[-\frac{1}{m^2 s_0} + \ln \frac{1}{\gamma m^2 s_0} + 1 \right] (\bar{\psi}(x) \gamma_5 \psi(x))^2. \quad (9')$$

Hence for the density of the vacuum current we obtain

$$\langle j(x) \rangle_{\text{vac}} = \frac{\delta L^{(1)}}{\delta \bar{\psi}(x)} = -\lambda \bar{\psi}(x) \gamma_5 \psi(x) \gamma_5 \psi(x), \quad (10)$$

where the constant of nonlinear interaction

$$\lambda = \frac{m^2}{4\pi^2} \frac{g^2}{\hbar c} \left[-\frac{1}{m^2 s_0} + \ln \frac{1}{\gamma m^2 s_0} + 1 \right] \quad (11)$$

also has the dimension of the Fermi constant g_F like our $\underline{g}$ (γ is Euler's constant 1.78 ...).

By substituting (10) in (4) we obtain the desired nonlinear equation for the field $\psi(x)$:

$$(-i \gamma_\mu \partial_\mu + \lambda \bar{\psi}(x) \gamma_5 \psi(x) \gamma_5 + m) \psi(x) = 0$$

or

$$(-i \gamma_\mu \partial_\mu - \lambda \bar{\psi}(x) \psi(x) + m) \psi(x) = 0. \quad (12)$$

This is the nonlinear equation of the Dirac field with the additional nonlinear term

$$\lambda [\bar{\psi}(x) \psi(x)] \psi(x),$$

whose form was previously shown by one of us [2] and which has recently been investigated by Heisenberg and others.

The evaluation of λ on the assumption $\sqrt{s_0} \approx 10^{-14}$ with $\underline{g} \approx g_F$, according to the concept of a universal interaction between any two pairs of Fermi spinor particles gives a value which is smaller but close in order of magnitude to the Fermi constant $\lambda \approx 10^{-48} \text{ erg} \cdot \text{cm}^3$ (concerning the choice of a cutoff parameter see [3]). From the point of view of a unitary field theory [4], which must be based on a nonlinear equation, it is reasonable to eliminate the mass term entirely, and to explain the nonlinear term in χ not as being induced by the vacuum, as is done in the present paper, but as having a certain "bare particle" character, and leaving open the question of the significance of χ until comparison with experiment can be made.

Thus it must be emphasized that with each particle of non-vanishing rest mass the quantum theory of the vacuum necessarily associates an infinite field mass, an infinite number of field charges (i.e., constants of interaction) and an infinite constant of nonlinear self-action which can be renormalized only because of the presence of suitable "bare particle" terms possessing mass, charge and nonlinearity.

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ATOMIC NICKEL CENTERS IN NaCl-Ni PHOSPHORS

M. L. Kats and B. Z. Semenov

(Presented by Academician G. S. Landsberg June 30, 1955)

NaCl-Ni phosphors have been studied by I. A. Parfianovich [1], who used rock salt crystals into which the activator was introduced by electrolysis from a nickel anode at a temperature somewhat lower than the melting point of the crystal. It was established that selective absorption by the activator appears in these crystals in the form of a characteristic band with a maximum at 246 m μ . Furthermore, the light from a mercury arc passing through a filter with a transmission band from 330 to 400 m μ excites in the electrochemically activated NaCl-Ni phosphors a brilliant orange-red fluorescence which they ascribed to the presence of nickel in an ionic state. If a NaCl-Ni phosphor is colored additively by electrons from a sodium cathode using the method of Artsibashev, certain regions of the crystals will be colored with a brownish-yellow light which is due, as was shown by K. N. Pogodaev [2], to absorption by colloidal particles of nickel with an absorption maximum at about 375 m μ . On the basis of Mie's theory the method of M. V. Savostyanova [3] was used to calculate the absorption curves of colloidal particles of nickel in the Ni-NaCl system, and it was shown that for certain dimensions of the particles the theoretically calculated curves agree with experimental curves [2]. In electrochemically activated NaCl-Ni phosphors irradiated with X-rays an afterglow is observed which agrees in spectral composition with the fluorescence of these phosphors.

I. A. Parfianovich suggested that as a result of X-irradiation there is additional ionization of the ions of the activator which then, upon recombination with electrons freed from trapping levels, make a transition to the ground state emitting orange-red light. However, V. L. Levshin considers [4] that the role of the nickel in NaCl-Ni phosphors has not been fully cleared up, just as in the case of many other crystalline phosphors, since a complete explanation of the role of the activator is connected with the fundamental and least studied problem of the formation of luminescent centers.

Some possibilities in the study of this problem in the case of alkali halide phosphors are offered by the investigations of the changes which appear in optical processes under the effect of X-rays. This method of investigation, which we have previously used [5] to study luminescent centers in NaCl-Ag and NaCl-Cu phosphors, enables one to obtain fundamentally new data on absorption and luminescence centers and on the role of the activator in the processes of electron localization in NaCl-Ni phosphors.

Figure 1 shows the absorption curve which we obtained for a NaCl-Ni phosphor grown from a melt (curve b). For comparison the same figure shows the absorption curve a of the same kind of phosphor in which the nickel was introduced electrochemically at a high temperature [1]. The ordinate axis gives values of the absorption coefficient in arbitrary units. From Fig. 1 it can be seen that the maxima in both cases coincide at 246 m μ . However in the long wave region the curves differ considerably. In the case of phosphors grown from a melt the absorption is terminated at about 300 to 320 m μ , whereas in the electrochemically activated NaCl-Ni phosphors the decline of the absorption curve in the long wave region extends right up the visible portion of the spectrum.

From data in the literature it is known [6] that at not too high concentrations the activator is incorporated into the lattice of alkali halide crystals by a process of isomorphic substitution, even when the original salts were nonisomorphic. Therefore it is reasonable to assume that at the comparatively low concentrations that are present in our case ($\sim 10^{-3}$ g/g) the nickel also enters the lattice by replacing cations of the alkali metal in lattice sites. Moreover, it may also be assumed that like other divalent activators [7] such as Pb $^{++}$, Sn $^{++}$, Cd $^{++}$ or mixtures of rare earth metals [8], Ca $^{++}$, Sr $^{++}$ and Ba $^{++}$, nickel during the growth of the phosphor from a NaCl-NiCl $_2$ melt enters the lattice in the form of Ni $^{++}$ ions to which the absorption band at 246 m μ is evidently due.

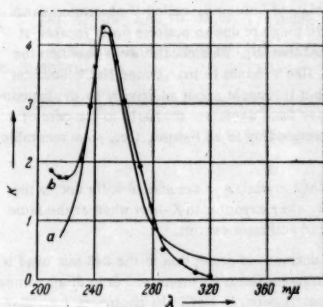


Fig. 1. Absorption curves of NaCl-Ni phosphors: a) activated electrochemically [1]; b) grown from a melt of NaCl + NiCl₂.

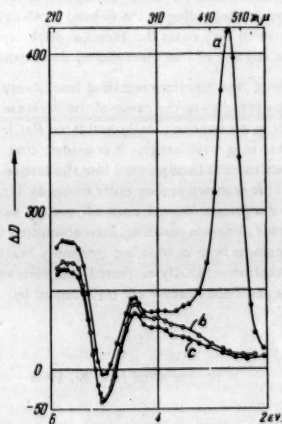


Fig. 2. Absorption curves of NaCl-Ni phosphors irradiated with X-rays: a) immediately after irradiation; b) after subsequent irradiation with 465 mμ light; c) after annihilation of the F-band and 68 hours in darkness.

are shown in Fig. 2. Along the ordinate axis are plotted the differences between the optical densities of two samples of NaCl-Ni of almost equal thickness cut from the same slab, only one of which was irradiated with X-rays. It can be seen from the figure that as a result of the X-rays there is a decline of absorption in the 246 mμ band that is caused by selective absorption of the activator. At the same time there appear new absorption bands whose maxima are at 216; 279; 310; ~350 and 465 mμ. The last is known to be due to F-Centers,

A second essential difference between electrochemically activated NaCl-Ni phosphors and those grown from a melt is the fact that the latter do not fluoresce at all when irradiated by filtered 330-400 mμ ultraviolet light, which in electrochemically activated phosphors excites orange-red fluorescence. There is nothing surprising in this if we consider that on the long wave side absorption by NaCl-Ni phosphors grown from a melt ceases at about 300-320 mμ. However, it appears that in these phosphors the orange-red fluorescence is not excited even under the action of shorter wave lengths. By shifting the crystal through the spectrum obtained by employing a quartz spectrograph, it can be shown that in NaCl-Ni phosphors grown from a melt the orange-red fluorescence is not excited even under light corresponding to the maximum of the selective absorption band of the activator.

In order to enhance the intensity of the exciting light, special filters were used to obtain individual lines and portions of the spectrum of the mercury arc: 365; 313; 280; 265 and 254 mμ simultaneously with the 248 mμ line. The latter correspond to the spectral region of strong absorption by NaCl-Ni.

It appeared that even in this case orange-red fluorescence is not excited in NaCl-Ni phosphors grown from a melt. It thus follows that the orange-red fluorescence observed in electrochemically activated NaCl-Ni phosphors [1] is not a characteristic of Ni⁺⁺ ions.

In NaCl-Ni phosphors grown from a melt bright orange-red fluorescence appears only after the phosphors are irradiated with X-rays. Then this fluorescence is excited just as in the case of electrochemically activated NaCl-Ni phosphors by long-wave ultraviolet light. Investigation of the spectral distribution of the excitation showed here also that light corresponding to the maximum of the selective absorption band of the activator (246 mμ) does not excite orange-red fluorescence even after exposing the phosphor to X-rays. From these experiments it follows that because of the X-rays certain changes take place in the activator centers of the phosphor as a result of which they are transformed from centers unable to fluoresce in the orange-red region into centers which do possess this property.

What takes place as a result of the X-rays? We may attempt to answer this question by investigating the changes which occur in the absorption spectra of NaCl-Ni phosphors subjected to X-rays. The results of measurements which were carried out by a differential method

which are formed by electrons localized at vacant halogen ion lattice sites.

We must put the band with a maximum at 216 m μ in the class of hole or so-called V absorption bands which are observed in pure NaCl crystals exposed to X-rays, where they are due to positive holes located at defects of the crystal structure, that is, vacant sites of alkali metal ions [9]. This conclusion is based on the strong similarity of the 216 m μ band with V-bands. The former, like V-bands in inactivated NaCl, does not appear even in NaCl-Ni phosphors if the coloration of the phosphor is brought about additively by the introduction of electrons from the outside. Furthermore, absorption in the above band decreases similarly to the case of absorption in V-bands when the phosphor is bleached by light corresponding to an F-band, i.e., upon recombination of free electrons with localized holes.

The 279, 310 and 350 m μ bands are not observed in pure NaCl crystals and are undoubtedly due to the activator which has been modified by X-rays, for they appear only after exposure to X-rays when at the same time there is a decrease of absorption in the band due to unchanged activator centers.

On the basis of the usual ideas [1], it might appear that the decrease of absorption in the 246 m μ band is caused by ionization of the activation centers. However, this interpretation is incorrect and is in disagreement with experimental data. Our experiments show that opposite processes occur in NaCl-Ni phosphors, i.e., that activator centers do not release but, on the contrary, capture electrons. This is shown by the following experimental evidence. If a NaCl-Ni phosphor, after exposure to X-rays, is irradiated with light corresponding to the F-band, the 246 m μ band is not revived; absorption in it does not increase as might have been expected from the recombination of electrons freed from F-levels with activator centers thus producing a 246 m μ band. When a NaCl-Ni phosphor after X-ray irradiation is irradiated with light absorbed in the F-band, absorption in the 246 m μ band decreases even further (curve b, Fig. 1). But since energy corresponding to the F-band (2.65 eV) cannot induce ionization of the center, for which about 5 eV is required, in both cases the decrease of absorption in the 2.46 m μ band is due not to ionization of the centers but to capture of free electrons by these centers.

Thus activator centers in the form of Ni⁺⁺ ions with the capture of free electrons resulting from X-rays are transformed into singly ionized and atomic nickel centers. Similar centers are the cause of the decrease on the long wave side of the selective absorption band of the activator in electrochemically activated NaCl-Ni phosphors and of orange-red fluorescence that is excited by light in that long wave range. It is evident that when NaCl crystals are activated with nickel electrochemically the activator is incorporated into the lattice not only in the form of ions but partially in the form of atomic centers. This is shown among other things by the data obtained by I. A. Parfianovich and S. A. Shipitsin [10], and up to the present has not been adequately explained. These authors have established that the weak brownish-yellow coloration resulting from absorption by colloidal nickel particles may be obtained not only by introducing electrons from outside but by simply heating a NaCl-Ni phosphor to 500°. Hence it follows immediately that in electrochemically activated phosphors some portion of the activators is actually present as atomic nickel centers, which upon heating are transformed by coagulation into colloidal particles.

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TEMPERATURE HYSTERESIS OF THE GALVANOMAGNETIC EFFECT

L. V. Kirenskii and V. F. Ivlev

(Presented by Academician A. B. Shubnikov November 6, 1954)

Magnetic temperature hysteresis means that in a constant magnetic field the magnetic characteristics of a ferromagnetic material depend on the previous temperature history of the specimen.

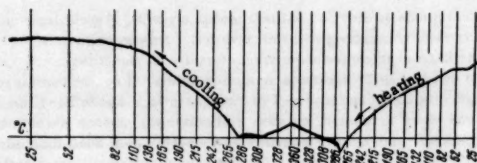


Fig. 1. Dependence of the galvanomagnetic effect on temperature during heating and cooling in a field of 39 oerst.

In particular such an important magnetic characteristic as the intensity of magnetization is not a unique function of the temperature for a given strength of the magnetic field. Such temperature hysteresis phenomena can be explained in two ways: 1) by the presence of structural changes in heterogeneous alloys; 2) by the presence of various factors which determine the practical magnetization curve in the region of domain boundary displacements. A beginning of a systematic study of the temperature hysteresis of magnetization was made in the work of I. A. S. Shur and V. I. Drozhzhina [3] who demonstrated the presence of this phenomenon in specimens of nickel and of silicon iron.

They showed that magnetic temperature hysteresis takes place only in the region of boundary displacement, i.e., in the steep portion of the curve of magnetization. With a large field and a transition to the rotation process, magnetic temperature hysteresis disappears.

Temperature hysteresis of magnetization must belong to temperature hysteresis in a number of other magnetic phenomena directly connected with the magnetization of a ferromagnet. Indeed L. V. Kirensky and A. Ia. Vlasov have observed temperature hysteresis of the magnetostriction of nickel [4].

The present article describes the magnetic temperature hysteresis of the galvanomagnetic effect which was first observed by the authors, i.e., the change of electric resistance when the specimen is placed in an external magnetic field.

The sample studied was a nickel wire of 0.05 mm diameter and 20 cm length. A balancing method was used, where the balancing specimen in another arm of the bridge was the same kind of nickel specimen wound in a spiral with the turns perpendicular to the magnetic field. Such use of a balancing specimen was very convenient, as it automatically excluded the factor of temperature dependence of the electric resistance. Indeed, in the absence of a magnetic field, a change of temperature all the way from room temperature to the Curie point did not disturb the balance of the bridge.

Thus, in a magnetic field, a change of temperature will clearly determine only the galvanomagnetic effect at different temperatures, but a cyclical change of temperature will reveal magnetic temperature hysteresis of the galvanomagnetic effect if the value of the latter depends on the previous temperature history of the specimen,

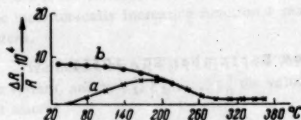


Fig. 2. Temperature hysteresis of the galvanomagnetic effect in a field of 12 oerst.
a) Heating; b) cooling.

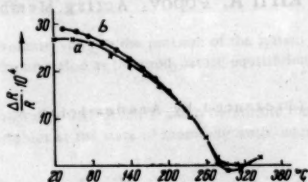


Fig. 3. Temperature hysteresis of the galvanomagnetic effect in a field of 39 oerst.
a) Heating; b) cooling.

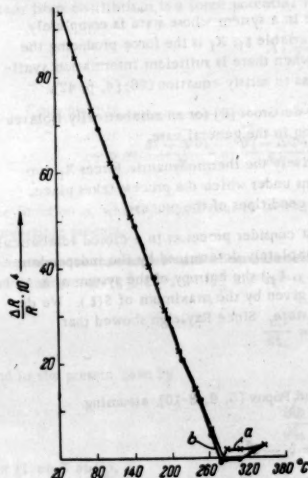


Fig. 4. Temperature hysteresis of the galvanomagnetic effect in a field of 160 oerst.
a) Heating; b) cooling.

The research employed automatic photographic recording of the phenomenon on a film in the form of a magnetogram. Figure 1 shows a magnetogram of the galvanomagnetic effect taken in a field of 39 oerst. from room temperature to the Curie point (heating) and then from the Curie point to room temperature (cooling). Vertical lines in the magnetogram intersect the horizontal axis at 25, 52, 82, 110, 138, 165, 190, 215, 242, 265, 286, 308, 328 and 360°, and decreasing in the same order.

As can be seen from the magnetogram, the values of the galvanomagnetic effect during heating do not agree in general with the values of this effect during cooling for identical temperatures, which undoubtedly demonstrates the existence of temperature hysteresis of the galvanomagnetic effect. Figures 2, 3, and 4 show magnetograms obtained in fields of 12, 39 and 160 oerst during heating from room temperature to the Curie point and subsequent cooling from the Curie point to room temperature. There is no doubt of the existence of temperature magnetic hysteresis of the galvanomagnetic effect.

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ON A SINGLE BASIC EQUATION IN THE THEORY OF IRREVERSIBLE THERMODYNAMICAL PROCESSES

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(Presented by Academician N. N. Andreev September 20, 1955)

K. Denbigh, on the basis of a discussion by W. Thomson, has derived the equation (Equation (26) in reference [4])

$$\dot{\theta} = I_1 X_1 + I_2 X_2 + \dots + I_n X_n,$$

where $\dot{\theta}$ is the rate of increase of entropy in various natural processes in a system whose state is completely defined by the variables ξ_i ($i = 1, 2, \dots, n$); I_i is the flux of the variable ξ_i ; X_i is the force producing the flux I_i . He notes, however, that these equations have meaning only when there is sufficient information available concerning the forces X_i , and that the forces must be chosen so as to satisfy equation (26) [4, p. 42].

This equation, which was derived earlier by Onsager [1] and by de Groot [2] for an adiabatically isolated system, is inapplicable, as we shall see, without considerable limitation in the general case.

In order to prove this equation it is necessary to determine precisely the thermodynamic forces X_i corresponding to the choice of independent variables ξ_i and the conditions under which the process takes place. The forces X_i cannot be chosen independently of each other or of the conditions of the process.

1. A process in an adiabatically isolated system. We shall first consider processes in a closed adiabatically isolated system whose thermodynamic state at a given instant is completely determined by the independent variables $\xi_1, \xi_2, \dots, \xi_n$, and we shall denote by $S(\xi) = S(\xi_1, \xi_2, \dots, \xi_n)$ the entropy of the system as a function of the variables ξ . The state of equilibrium of the system is given by the maximum of $S(\xi)$. We denote by $\xi_1^0, \xi_2^0, \dots, \xi_n^0$ the values of the variables in the equilibrium state. Since Rayleigh showed that

$$(-\Delta S) = S(\xi) - S(\xi^0)$$

can be considered a function of the forces, Onsager [1], de Groot [2] and Popov [5, 6, 8-10], assuming

$$X_i = \frac{\partial(-\Delta S)}{\partial \xi_i} = \frac{\partial(-\Delta S)}{\partial x_i}, \text{ where } x_i = \xi_i - \xi_i^0,$$

obtained

$$\begin{aligned} -\dot{\theta} &= \frac{d(-\Delta S)}{dt} = \frac{\partial(-\Delta S)}{\partial x_1} \frac{dx_1}{dt} + \dots + \frac{\partial(-\Delta S)}{\partial x_n} \frac{dx_n}{dt} = \\ &= X_1 I_1 + X_2 I_2 + \dots + X_n I_n, \end{aligned}$$

where $I_i = d\xi_i/dt = dx_i/dt$. This is Denbigh's equation (26) written in our notation.

In the general case the state of thermodynamic equilibrium is determined by the appropriate characteristic function.

2. Isothermic isobaric processes. We now consider thermodynamic processes in a closed system which exchanges heat with the surrounding medium at constant temperature and pressure. The characteristic function determining the state of thermodynamic equilibrium and the irreversible processes in this system are given by [3]

$$\Phi = S - \frac{U + pV}{T},$$

where S is the entropy, U is the internal energy, V is the volume and p is the pressure of the system [11-13]. The monotonically increasing function Φ reaches its maximum value at thermodynamic equilibrium of the system.

We denote by $\xi_1, \xi_2, \dots, \xi_n$ the variables which together with given T and p determine the state of the system, and by $\xi_1^0, \xi_2^0, \dots, \xi_n^0$ the values of these variables at the state of thermodynamic equilibrium. We assume

$$\Delta\Phi = \Phi(\xi) - \Phi(\xi^0), \quad x_i = \xi_i - \xi_i^0,$$

so that $x_i = 0$ corresponds to the state of equilibrium. Here $\Delta\Phi$, as a measure of the degree of deviation of the system from equilibrium, is a force potential function, so that [11-13]

$$X_i = \frac{\partial(-\Delta\Phi)}{\partial\xi_i} = -\frac{\partial(\Delta\Phi)}{\partial\xi_i}.$$

Consequently

$$-\dot{\Phi} = \frac{d(-\Delta\Phi)}{dt} = \frac{\partial(-\Delta\Phi)}{\partial x_1} \frac{dx_1}{dt} + \dots + \frac{\partial(-\Delta\Phi)}{\partial x_n} \frac{dx_n}{dt} = X_1 I_1 + \dots + X_n I_n.$$

The function Φ , which is analogous to Denbigh's function (26), determines the rate of increase of Φ in a natural irreversible process.

We note that in the case of an adiabatically isolated process the state of thermodynamic equilibrium, and consequently $\xi_1^0, \xi_2^0, \dots, \xi_n^0$ and $x_i = \xi_i - \xi_i^0$, are given by the equations

$$\frac{\partial S}{\partial \xi_1} = \frac{\partial S}{\partial \xi_2} = \dots = \frac{\partial S}{\partial \xi_n},$$

and in the present case by

$$\frac{\partial \Phi}{\partial \xi_1} = \frac{\partial \Phi}{\partial \xi_2} = \dots = \frac{\partial \Phi}{\partial \xi_n}$$

for given T and p .

3. We also consider the case of isothermic processes in which the system exchanges heat with the surrounding medium and assuming that the external work is zero, as occurs in very many chemical processes [3]. Thermodynamic equilibrium is here represented by the minimum of the characteristic function

$$F = U - TS.$$

We denote by $\xi_1, \xi_2, \dots, \xi_n$ the independent variables determining for a given T the thermodynamic state of the system, and by $\xi_1^0, \xi_2^0, \dots, \xi_n^0$ their values for stable equilibrium; we let $x_i = \xi_i - \xi_i^0$ so that $x_i = 0$ at stable equilibrium. The function

$$\Delta F(\xi) = F(\xi) - F(\xi^0)$$

measures the degree of deviation of the system from the state of stable equilibrium. The forces X_i are here

$$X_i = -\frac{\partial \Delta F}{\partial x_i},$$

so that

$$f = \frac{d(\Delta F)}{dt} = \frac{\partial(\Delta F)}{\partial x_1} \frac{dx_1}{dt} + \dots + \frac{\partial(\Delta F)}{\partial x_n} \frac{dx_n}{dt} = X_1 I_1 + \dots + X_n I_n;$$

this is analogous to Denbigh's equation (26).

Thus in each concrete case we must start with the appropriate characteristic function, which in the present case is a force potential function, without forgetting that ξ_i^0 and consequently $x_i = \xi_i - \xi_i^0$ differ according to the conditions under which the irreversible process takes place. The forces X_i are determined by the appropriate potential function.

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SOLUTION OF THE PROBLEM OF THE MOTION OF THREE NUCLEONS IN THE LIMITING CASE OF SHORTRANGE FORCES

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(Presented by Academician L. D. Landau June 24, 1955)

The nuclear interaction of two nucleons is determined by a potential well $U(\rho)$ of small radius $r_0 \sim 10^{-13}$ cm and great depth $U_0 \sim \hbar^2/Mr_0^2 \sim 20$ Mev (M is the mass of a nucleon). If the energy of relative motion of the nucleons E is small ($E < U_0$, $\lambda = \hbar/\sqrt{ME} > r_0$), the problem of motion is solved in the form of a power series in r_0 , the range of the forces; the parameters of the expansion are the quantities r_0/λ and $\alpha r_0 < 1$, where $1/\alpha$ is the scattering length or radius of the bound state.

In zero approximation, corresponding to $r_0 \rightarrow 0$, the wave function satisfies the condition

$$\left\{ \frac{d(\rho \Psi)}{d\rho} \right\}_{\rho=0} = -\alpha (\rho \Psi)_{\rho=0}. \quad (1)$$

Here the interaction between the nucleons is characterized by the value of the constant α , i.e., the properties of the system of two nucleons do not depend on the details of the behavior of the potential $U(\rho)$, which may be chosen to have the form

$$U(\rho) = -4\pi \frac{\hbar^2}{M} \rho \delta(\rho). \quad (2)$$

Condition (1) with the potential (2) in Schrödinger's equation enables us to obtain the wave function outside the range of the forces.

The problem of the motion of three nucleons permits us to use a similar power series in r_0 if $r_0/\lambda < 1$ and $\alpha r_0 < 1$, where $\lambda = \hbar/\sqrt{ME}$, E being the energy of the system. This expansion may be utilized to examine the problem of the scattering of low-energy neutrons (up to 10 or 15 Mev) by deuterons, and also to construct the wave functions of the H^3 and He^3 nuclei.

We have given below the zero approximation of the theory for $r_0 \rightarrow 0$, $U_0 \sim \hbar^2/Mr_0^2 \rightarrow \infty$ (this limiting case is analogous to Bethe and Peierl's theory of the deuteron [1]). Here the three-nucleon wave function $\Psi(r_1, r_2, r_3)$ for $\rho_{ik} = r_i - r_k \rightarrow 0$, ($i, k = 1, 2, 3; i \neq k$), as can easily be seen*, satisfies the same boundary condition (1) as in the case of two particles and the properties of the system are independent of the behavior of $U(\rho)$ which, just as in the case of two bodies, may be given the form (2).

For simplicity we shall first neglect the spin of the nucleons (and the Coulomb interaction), considering them to be three identical particles. The coordinates of the nucleons in the center of mass systems are determined by two vectors (Jacobi's coordinates) $\vec{\rho}_{23} = r_2 - r_3$, and $\vec{\rho}_1 = \frac{r_1 - r_2}{2} - r_3$, where the wave function must be symmetric with respect to r_1, r_2, r_3 , i.e., $\Psi(\vec{\rho}_{23}, \vec{\rho}_1) = \Psi(\vec{\rho}_{12}, \vec{\rho}_3) = \Psi(\vec{\rho}_{31}, \vec{\rho}_2)$ (where $\vec{\rho}_{12}, \vec{\rho}_3 = \vec{\rho}_{21}, \vec{\rho}_2$ are determined like $\vec{\rho}_{23}$, and $\vec{\rho}_1$) and satisfies Schrödinger's equation:

* The value of the logarithmic derivative of the wave function at r_0 is determined by the behavior of the function inside the range of action of the forces. For $r_0 \rightarrow 0$, $U_0 \rightarrow \infty$, the latter depends exclusively on the potential $U(\rho)$ and is not changed by the presence of a third particle, which is found with only small probability in the region of interaction between two closely-positioned particles, under these conditions.

$$\left\{ -\frac{\hbar}{M} \left(\nabla_{\rho_{22}}^2 + \frac{3}{4} \nabla_{\rho_1}^2 \right) + U(\rho_{22}) + U(\rho_{12}) + U(\rho_{11}) \right\} \Psi = E \Psi,$$

which, with the aid of the Green's function

$$G_E = \frac{1}{(2\pi)^3} \int e^{ik(\vec{\rho}_1 - \vec{\rho}_1')} \frac{e^{-\gamma_k |\vec{\rho}_{22} - \vec{\rho}_{22}'|}}{|\vec{\rho}_{22} - \vec{\rho}_{22}'|} dk$$

(where $\gamma_k = \sqrt{3/4 k^2 - ME/\hbar^2}$, if the expression under the radical is positive, and $\gamma_k = -i \sqrt{ME/\hbar^2 - 3/4 k^2}$ in the contrary case) is easily transformed into the integral equation

$$\Psi(\vec{\rho}_{22}, \vec{\rho}_1) = -\frac{M}{4\pi\hbar^2} \int G_E(\vec{\rho}_{22} - \vec{\rho}_{22}', \vec{\rho}_1 - \vec{\rho}_1') [U(\rho_{22}') + U(\rho_{12}') + U(\rho_{11}')] \Psi(\vec{\rho}_{22}', \rho_1') d\vec{\rho}_{22}' d\rho_1'.$$

The Green's function G_E satisfies the equation

$$\left(\nabla_{\rho_{22}}^2 + \frac{3}{4} \nabla_{\rho_1}^2 + \frac{ME}{\hbar^2} \right) G_E = -4\pi \delta(\vec{\rho}_{22} - \vec{\rho}_{22}') \delta(\rho_1 - \rho_1')$$

and the boundary conditions at infinity corresponding to the above choice of the sign of γ_k . Substituting into the integral equation $U(\rho)$ from (2) and using the designation $\chi_1(\vec{\rho}_1) = \lim_{\rho_{22} \rightarrow 0} \{ \rho_{22} \Psi(\vec{\rho}_{22}, \rho_1) \}$, we obtain after

some not too complicated transformations • the following expression for the Fourier components of the function Ψ :

$$\begin{aligned} \int e^{-ik\vec{\rho}_1} \Psi(\vec{\rho}_{22}, \vec{\rho}_1) d\vec{\rho}_1 &= F(\vec{\rho}_{22}, k_1) = \\ &= \frac{e^{-\gamma_k \rho_{22}}}{\rho_{22}} \chi(k) + \int \frac{8\pi \cos(k' + 1/2 k, \rho_{22})}{k^2 + k'^2 + kk' - ME/\hbar^2 - i\tau} \chi(k') \frac{dk'}{(2\pi)^3}, \end{aligned} \quad (3)$$

where $\chi(k) = \int e^{-ik\vec{\rho}_1} \chi_1(\vec{\rho}_1) d\vec{\rho}_1$. By τ we denote the real positive number $\tau \rightarrow 0$, which is introduced for the correct passage around the pole in the integral of (3) which appears if $E > 0$.

• For example, for the Fourier components of the second term in the integral equation we have

$$\begin{aligned} \int e^{-ik\vec{\rho}_1} \frac{e^{-\gamma_k |\vec{\rho}_{22} - \vec{\rho}_{22}'|}}{|\vec{\rho}_{22} - \vec{\rho}_{22}'|} \delta(\rho_{22}') \chi_1(\vec{\rho}_2') d\vec{\rho}_{22}' d\rho_2' &= \int \chi(k') \frac{dk'}{(2\pi)^3} \int \frac{e^{-\gamma_k |\vec{\rho}_{22} - \vec{\rho}_2'|}}{|\vec{\rho}_{22} - \vec{\rho}_2'|} \times \\ &\times e^{i(k' + 1/2 k) \cdot \vec{\rho}_2'} d\vec{\rho}_2' = \int \frac{4\pi e^{-i(k' + 1/2 k, \vec{\rho}_{22})}}{k^2 + k'^2 + kk' - ME/\hbar^2 - i\tau} \chi(k') \frac{dk'}{(2\pi)^3}, \end{aligned}$$

so that for

$$\rho_{22} \rightarrow 0 \quad \vec{\rho}_{22}' = -\vec{\rho}_2', \quad \rho_2' = -1/2 \rho_2', \quad \text{and} \quad \int \frac{e^{-\gamma_k \rho}}{\rho} e^{i(k' + 1/2 k) \cdot \vec{\rho}} d\vec{\rho} = \frac{4\pi}{\gamma_k^2 + (k' + 1/2 k)^2 - i\tau} = \frac{4\pi}{k^2 + k'^2 + kk' - ME/\hbar^2 - i\tau}.$$

From (1) it follows that

$$\left\{ \frac{d}{d\rho} [\rho F(\vec{\rho}, \mathbf{k})] \right\}_{\rho=0} = -\alpha (\rho F(\vec{\rho}, \mathbf{k}))_{\rho=0};$$

substituting (3) here we obtain the integral equation for $\chi(\mathbf{k})$:

$$\left(\alpha - \sqrt{\frac{3}{4} k^2 - \frac{ME}{\hbar^2}} \right) \chi(\mathbf{k}) + \int \frac{8\pi \chi(\mathbf{k}')}{k^2 + k'^2 + \mathbf{k}\mathbf{k}' - ME/\hbar^2 - i\tau} \frac{d\mathbf{k}'}{(2\pi)^3} = 0, \quad (4)$$

the solution of which in accordance with (3) determines the wave function of the system.

Thus in the above approximation the three body problem reduces to the solution of the integral equation (4); for the S-state of the system, or for a state with any other definite value of the moment, the three-dimensional equation (4) reduces to a one-dimensional equation which can be solved numerically. The above method was suggested by L. D. Landau.

We now consider the problem of the scattering of one of the particles by a bound state of the other two; such a state exists if $\alpha > 0$, and is described by the wave function $\varphi_0(\rho) = \sqrt{\frac{\alpha}{2\pi}} \frac{e^{-\alpha\rho}}{\rho}$; the energy of the

bound state is $\epsilon = -\frac{\hbar^2 \alpha^2}{M}$. Here $E = \frac{\hbar^2 k_0^2}{2\mu} - \frac{\hbar^2 \alpha^2}{M}$, $\mu = \frac{2}{3} M$, i.e., $\frac{ME}{\hbar^2} = -(\alpha^2 - \frac{3}{4} k_0^2)$, where $\hbar k_0$ is the momentum of the relative motion of the incident particle. For large $\vec{\rho}_1$, (or $\vec{\rho}_2$, or $\vec{\rho}_3$) the wave function Ψ must go over into the product of the functions φ_0 by the sum of the plane and scattered waves. This requirement will be satisfied if (4) has a solution of the form

$$\chi(\mathbf{k}) = \sqrt{\frac{\alpha}{2\pi}} \left[(2\pi)^{3/2} \delta(\mathbf{k} - \mathbf{k}_0) + \frac{4\pi a(\mathbf{k}, \mathbf{k}_0)}{k^2 - k_0^2 - i\tau} \right],$$

where the amplitude of elastic scattering, defined as usual by $\underline{a}(\theta) = \lim_{k^2 \rightarrow k_0^2} \frac{k^2 - k_0^2}{4\pi} \int \varphi_0^*(\vec{\rho}_{23}) F(\vec{\rho}_{23}, \mathbf{k}) d\vec{\rho}_{23}$,

agrees for this choice of $\chi(\mathbf{k})$ with $\underline{a}(\mathbf{k}', \mathbf{k}_0)$; $|k_0| = |\mathbf{k}_0|$, $\cos \theta = \mathbf{k}_0 \mathbf{k}' / k_0 k'$. Substituting $\chi(\mathbf{k})$ in (4) we obtain an inhomogeneous integral equation for $\underline{a}(\mathbf{k}, \mathbf{k}_0)$:

$$\begin{aligned} \frac{\frac{3}{2} a(\mathbf{k}, \mathbf{k}_0)}{\sqrt{\frac{3}{4} k^2 - ME/\hbar^2} + \alpha} &= \frac{1}{k^2 + k_0^2 + \mathbf{k}\mathbf{k}_0 - ME/\hbar^2 - i\tau} + \\ &+ 4\pi \int \frac{a(\mathbf{k}', \mathbf{k}_0)}{(k^2 - k_0^2 - i\tau)(k^2 + k'^2 + \mathbf{k}\mathbf{k}' - ME/\hbar^2 - i\tau)} \frac{d\mathbf{k}'}{(2\pi)^3}, \end{aligned} \quad (5)$$

the solution of which determines the continuous spectrum wave function and the scattering amplitude. In the limiting case of the scattering of zero energy particles ($k_0 = 0$, $\alpha^2 = -ME/\hbar^2$) (5) has a real solution $\underline{a}(\mathbf{k}, 0) = a_0(\mathbf{k})$, independent of θ (S-state) and satisfying the equation

$$\frac{\frac{3}{2} a_0(\mathbf{k}, \mathbf{k}_0)}{\sqrt{\frac{3}{4} k^2 - ME/\hbar^2} + \alpha} = \frac{1}{k^2 + \alpha^2} + \frac{1}{2\pi} \int_0^\infty \frac{a_0(k')}{kk'} \ln \frac{k^2 + k'^2 + \mathbf{k}\mathbf{k}' + \alpha^2}{k^2 + k'^2 - \mathbf{k}\mathbf{k}' + \alpha^2} dk', \quad (6)$$

which it is easy to solve numerically.

The theory is easily generalized to include spin and isotopic spin for application to the problem of neutron scattering by deuterons. Here two states are possible: with spin $1/2$ or $3/2$.

If we assume isotopic invariance of nuclear forces in the above approximation, the wave function corresponding to spin $1/2$ is determined [2] by two functions analogous to $\gamma(k)$, and the wave function for spin $3/2$ by one function. In the limiting case $k_0 = 0$, $-ME/\hbar^2 = \alpha_1^2$ the analog of equation (6) will be:

for $s = 1/2$:

$$\begin{aligned} & \frac{\sqrt{1/4 k^2 + \alpha_1^2} - \alpha_1}{k^2} a_{1/2}(k) = \\ & = \frac{1/2}{k^2 + \alpha_1^2} + \frac{1}{\pi} \int_0^\infty [\frac{1}{2} a_{1/2}(k') + \frac{1}{2} b_{1/2}(k')] \frac{1}{kk'} \lg \frac{k^2 + k'^2 + kk' + \alpha_1^2}{k^2 + k'^2 - kk' + \alpha_1^2} dk'; \quad (7a) \\ & \frac{\sqrt{1/4 k^2 + \alpha_1^2} - \alpha_1}{k^2} b_{1/2}(k) = \\ & = \frac{1/2}{k^2 + \alpha_1^2} - \frac{1}{\pi} \int_0^\infty [\frac{1}{2} a_{1/2}(k') + \frac{1}{2} b_{1/2}(k')] \frac{1}{kk'} \lg \frac{k^2 + k'^2 + kk' + \alpha_1^2}{k^2 + k'^2 - kk' + \alpha_1^2} dk'; \end{aligned}$$

for $s = 3/2$:

$$\begin{aligned} & \frac{\sqrt{1/4 k^2 + \alpha_1^2} - \alpha_1}{k^2} a_{3/2}(k) = \\ & = \frac{-1}{k^2 + \alpha_1^2} - \frac{1}{\pi} \int_0^\infty \frac{a_{3/2}(k')}{kk'} \lg \frac{k^2 + k'^2 + kk' + \alpha_1^2}{k^2 + k'^2 - kk' + \alpha_1^2} dk', \quad (7b) \end{aligned}$$

where $1/\alpha_1 = 25 \cdot 10^{-13}$ cm and $1/\alpha_2 = 4.32 \cdot 10^{-13}$ cm are the scattering lengths respectively for the singlet and triplet states of two nucleons.

The values of $\alpha_{1/2}(0)$ and $\alpha_{3/2}(0)$ give the corresponding scattering amplitudes of neutrons by deuterons. A numerical integration of (7a) and (7b) gives: $f_{1/2} = -a_{1/2}(0) = 0.30 \cdot 10^{-12}$ cm, $f_{3/2} = -a_{3/2}(0) = 0.51 \cdot 10^{-12}$ cm. For the elastic scattering cross-section we then obtain $\sigma = 2.55$ barns, whereas from experiment [3] $\sigma = 3.4 \pm 0.06$ barns.

In the above-considered case, with $k_0 = 0$ the error of the zero approximation is associated with neglect of terms of the order of α_1^2 , i.e., the same error as in the deuteron theory of Bethe and Peierls.

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THE CLASSIFICATION OF NONLINEAR EQUATIONS AND OF RELATIVISTICALLY INVARIANT INTERACTIONS ACCORDING TO REPRESENTATIONS OF THE LORENTZ GROUP AND THE THEORY OF FUSION

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1. In an earlier paper [1] a classification was given of nonlinear relativistically invariant equations for Bose fields according to representations of the Lorentz group.

2. The method developed in [1] enables us to investigate all nonlinear spinor equations. Let us consider for example the nonlinear Dirac equation

$$\gamma_i^{\alpha\beta} \frac{\partial \psi_\beta}{\partial x_i} + \kappa \psi_\alpha + \lambda (\bar{\psi}_\beta \psi_\beta) \psi_\alpha = \eta_\alpha. \quad (1)$$

Using equations of references [2, 3] and introducing the functional $Z = \langle T e^{-i\alpha} \rangle_0^{-1}$, we write equation (1) and its conjugate in the form

$$\left\{ -i N_\alpha^{(i)} \frac{\partial}{\partial x_\alpha} + \Gamma_\alpha^{(i)} \frac{\partial}{\partial \eta_\alpha} + \bar{\Gamma}_\alpha^{(i)} \frac{\partial}{\partial \bar{\eta}_\alpha} - I^{(i)} \right\} \Psi = 0 \quad (i = 1, 2, 3, 4), \quad (2)$$

where

$$\begin{aligned} \Gamma_\alpha^{(i)} &= \begin{vmatrix} \Gamma_{1\alpha}^{(i)} & 0' \\ 0' & \Gamma_{2\alpha}^{(i)} \end{vmatrix}; & \bar{\Gamma}_\alpha^{(i)} &= \begin{vmatrix} \Gamma_{1\alpha}^{(i)} & 0' \\ 0' & \Gamma_{2\alpha}^{(i)} \end{vmatrix}; \\ \Gamma_{1\alpha} &= \begin{vmatrix} 0 & \lambda \delta_{1\alpha} & 0 & \dots & 0 \\ 0 & & & & \\ \delta_{1\alpha} & & 0 & & \\ \vdots & & & & \\ \delta_{4\alpha} & \dots & \dots & \dots & 0 \end{vmatrix}; & \Gamma_{2\alpha} &= \begin{vmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \delta_{1\alpha} & \delta_{4\alpha} \\ & & & \vdots \\ & & 0 & \\ 0 & \dots & \dots & 0 \end{vmatrix}; \\ I^{(i)} &= \begin{vmatrix} I_1^{(i)} & 0' \\ 0' & I_2^{(i)} \end{vmatrix}; & I_1^{(i)} &= \begin{vmatrix} \bar{\eta}_1 & 0 & i\kappa \delta_{11} & \dots & i\kappa \delta_{14} \\ & 1 & & & \\ 0 & & \ddots & & \\ & & & \ddots & \\ 0 & \dots & \dots & & 1 \end{vmatrix}; & I_2^{(i)} &= \begin{vmatrix} \eta_1 & 0 & i\kappa \delta_{11} & \dots & i\kappa \delta_{14} \\ & 1 & & & \\ 0 & & \ddots & & \\ & & & \ddots & \\ 0 & \dots & \dots & & 1 \end{vmatrix}; \end{aligned}$$

the $N_\alpha^{(i)}$ are matrices which do not have reciprocals and are given by the coefficients of the Dirac matrices. The multi-component function

$$\Psi = [Z H H_1 \dots H_4 Z H \bar{H}_1 \dots \bar{H}_4],$$

$$H_i = \frac{\partial}{\partial \eta_i}; \quad \bar{H}_i = \frac{\partial}{\partial \bar{\eta}_i}; \quad H = \frac{\partial H_i}{\partial \eta_i} = \frac{\partial \bar{H}_i}{\partial \bar{\eta}_i},$$

is transformed in accordance with the representation $R \sim \{ (01), (01), (01)(01), (-\frac{1}{2}, \frac{3}{2}), (\frac{1}{2}, \frac{3}{2}), (-\frac{1}{2}, \frac{3}{2}) \} = (\frac{1}{2}, \frac{3}{2})$. The matrices $N_\alpha^{(i)} \Gamma_\alpha^{(i)} \bar{\Gamma}_\alpha^{(i)}$ are given by the commutation relations resulting from the relativistic invariance of (1):

$$\begin{aligned}
[\Gamma_a^{(i)} I^{\mu}] &= - \sum_j S_{aj}^{\mu} \Gamma_j^{(i)} - \sum_l S_{l\beta}^{\mu} \Gamma_a^{(l)}, \\
[\bar{\Gamma}_a^{(i)} I^{\mu}] &= - \sum_j S_{aj}^{\mu} \bar{\Gamma}_j^{(i)} - \sum_\beta S_{\beta l}^{\mu} \bar{\Gamma}_a^{(l)}, \quad -g_{44} = g_{11} = g_{22} = g_{33}, \\
[N_\beta^{(i)} I^{\mu}] &= g_{\beta k} N_l^{(i)} - g_{\beta l} N_k^{(i)},
\end{aligned} \quad (3)$$

where I^{kl} and S^{kl} are infinitesimal operators belonging to the representations R and $(-1/2, 1/2) = (1/2, 1/2)$.

3. These considerations are also applicable to systems of equations which describe interacting fields. As an example we shall consider a system of equations which describe Fermi and Bose particles:

$$\begin{aligned}
\gamma_j^{ik} \frac{\partial \psi_k}{\partial x_j} + x \psi_i - \gamma_j^{ik} A_j \psi_k &= \eta_i; \\
\bar{\gamma}_j^{ik} \frac{\partial \bar{\psi}_k}{\partial x_j} - x \bar{\psi}_i - \bar{\gamma}_j^{ik} A_j \bar{\psi}_k &= \bar{\eta}_i; \quad (i = 1, 2, 3, 4) \\
(\square - m^2) A_i - \frac{ie}{2} [\bar{\psi}, \gamma_i \psi] - J_i &= 0,
\end{aligned} \quad (4)$$

where η_i and J_i are spinor and vector sources respectively.

Introducing Z and taking the functional derivatives of Z with respect to $\bar{\eta}_i$, $\bar{\eta}_i$ and J_i as new variables, we write (4) in matrix form:

$$\begin{aligned}
\left\{ N_a^{(i)} \frac{\delta}{\delta J_a} + B_k^{(i)} \frac{\delta}{\delta \eta_k} - \bar{B}_k^{(i)} \frac{\delta}{\delta \bar{\eta}_k} - I^{(i)} \right\} \Psi &= 0; \\
N_a^{(i)} &= \begin{vmatrix} \Gamma_a^{(i)} & 0 \\ 0 & P_a^{(i)} \end{vmatrix}; \quad B_k^{(i)} = \begin{vmatrix} L_k^{(i)} & 0 \\ 0 & R_k^{(i)} \end{vmatrix}; \quad \bar{B}_k^{(i)} = \begin{vmatrix} 0 & \bar{L}_k^{(i)} \\ 0 & \bar{R}_k^{(i)} \end{vmatrix}; \\
I_a^{(i)} &= \begin{vmatrix} 0 & \dots & 0 \\ \delta_{1a} & & \\ \vdots & 0 & \\ \delta_{4a} & \dots & 0 \end{vmatrix}; \quad P_a^{(i)} = \begin{vmatrix} -i(\square - m^2) \delta_{1a} & \dots & 0 \\ \vdots & 0 & \vdots \\ 0 & \dots & 0 \end{vmatrix}; \\
L_k^{(i)} &= \begin{vmatrix} -iD_{ik} & -\gamma_1^{ik} & \dots & \gamma_4^{ik} \\ \vdots & 0 & & \vdots \\ 0 & \dots & \dots & 0 \end{vmatrix}; \quad \bar{L}_k^{(i)} = \begin{vmatrix} -i\bar{D}_{ik} & -\bar{\gamma}_1^{ik} & \dots & -\bar{\gamma}_4^{ik} \\ \vdots & 0 & & \vdots \\ 0 & \dots & \dots & 0 \end{vmatrix}; \\
D_{ik} &= \gamma_j^{ik} \frac{\partial}{\partial x_j} + x \delta_{ik}; \\
I^{(i)} &= \begin{vmatrix} \Omega^{(i)} & 0 \\ 0 & \Gamma^{(i)} \end{vmatrix};
\end{aligned} \quad (5)$$

$$\Omega^{(i)} = \begin{vmatrix} \eta_i & & \\ & 1 & 0 \\ & 1 & \\ & 0 & 1 \\ & & & 1 \end{vmatrix}; \quad \bar{\Omega}^{(i)} = \begin{vmatrix} \eta_i & & \\ & 1 & 0 \\ & 1 & \\ & 0 & 1 \\ & & & 1 \end{vmatrix}; \quad T^{(i)} = \begin{vmatrix} J_i & & \\ & 1 & 0 \\ & 1 & \\ & 0 & 1 \\ & & & 1 \end{vmatrix};$$

$$\Psi = [ZH_1 \dots H_4 Z H_1 \dots H_4 Z K_1 \dots K_4 \bar{K}_1 \dots \bar{K}_4];$$

$$H_j = \frac{\delta Z}{\delta J_j}; \quad K_j = \frac{\delta Z}{\delta \eta_j}; \quad \bar{K}_j = \frac{\delta Z}{\delta \bar{\eta}_j} \quad (j = 1, 2, 3, 4);$$

$$R_k^{(i)} = \begin{vmatrix} 0 \dots 0 & -\frac{ie}{2} \gamma_{k1}^i & \dots & -\frac{ie}{2} \gamma_{k4}^i \\ \delta_{k1} & & & \\ \vdots & & & \\ \delta_{k4} & & & \\ 0 & & & \\ \vdots & & & \\ 0 & & & 0 \end{vmatrix}; \quad \bar{R}_k^{(i)} = \begin{vmatrix} 0 & \frac{ie}{2} \gamma_{k1}^i & \dots & \frac{ie}{2} \gamma_{k4}^i & 0 \dots 0 \\ 0 & & & & \\ 0 & & & & \\ 0 & & 0 & & \\ \delta_{k1} & & & & \\ \vdots & & & & \\ \delta_{k4} & & & & \\ \vdots & & & & \\ 0 & & & & \\ \delta_{k1} & & & & \end{vmatrix}.$$

Ψ is transformed in accordance with the representation $R \sim \{(01), (01), (01), (02), (02), (02), D, D\}$, where $D \sim \{(-1/2, 3/2), (1/2, 3/2)\}$. The matrices $N_\alpha^{(i)}$, $B_K^{(i)}$, $\bar{B}_K^{(i)}$ satisfy the commutation relations

$$[N_\alpha^{(i)}, K^k] = - \sum_j S_{\alpha j}^k N_j^{(i)} - \sum_n S_{i n}^k N_\alpha^{(n)}; \quad (6)$$

$$[B_p^{(i)}, K^k] = - \sum_j K_{p j}^k B_j^{(i)} - \sum_n K_{i n}^k B_p^{(n)}; \quad [\bar{B}_p^{(i)}, K^k] = - \sum_j K_{p j}^k \bar{B}_j^{(i)} - \sum_n K_{i n}^k \bar{B}_p^{(n)},$$

where K^{kl} and S^{kl} are operators of the representations D and (02) and 1^{kl} of the representation R .

A similar procedure is possible for the case of Fermi and Bose fields of arbitrary spin.

Thus the method described in [1] makes it possible to classify relativistically invariant interactions by representations of the Lorentz group. The interactions as well as the nonlinear terms are then interpreted as a new degree of freedom given by the invariants of the representation of the Lorentz group.

4. These results enable us to develop a method which is a generalization of de Broglie's method of fusion [4]. For this purpose the method of fusion is applied to the nonlinear spinor equations in a more general formulation given in [5]. We consider the fusion of Dirac's nonlinear equations as given by representations $R \sim \{(01)_2, D_4\}$ (the index of the representations shows how many times it is contained in R). In the usual notation the equation is of the form

$$T_{ij}^{\mu} \frac{\partial \psi_k}{\partial x_j} + \kappa \psi_i + S_{ip}^i T_{ki}^{\mu} \bar{\psi}_k \bar{\psi}_p = \eta_i, \quad (7)$$

where T_{kl}^{μ} and $S_{\nu p}^i$ are Clebsch-Gordan coefficients; i and ν are respectively the indices of representations D and $K \sim \{(01), (01), (02), (02), (-12), (12)\}$. According to [6] the equation of fusion will be given by the direct product $R \times R = \{(01)_4, D_{16}, K_{16}\}$ with the aid of the commutation relations (6). As has already been shown this representation gives in general a system of nonlinear equations which describe interacting fields of integral and half-integral spins. But it is evident that the equation of fusion is not unique. As in the case of

the linear equations (6) we have a family of equations corresponding to $R \times R_n$. In order to make the procedure unique it is necessary to impose supplementary physical conditions regarding the nature of the interaction. In a particular case the relations (6), where S^{kl} and K^{kl} belong to representations K and D , are satisfied by matrices of the type of (5) $\Sigma_K^{(l, \alpha)}$, $\bar{\Sigma}_K^{(l, \alpha)}$, $M_K^{(l, \alpha)}$.

After some rather cumbersome transformations the equation of fusion can be written in the form of a system of interacting nonlinear Dirac and Duffin equations with sources,

$$\begin{aligned} \gamma_\nu^{lk} \frac{\partial \psi_k}{\partial x_\nu} + x' \psi_l + T_{\beta n_1}^l S_{\alpha n_1}^{\beta} \dots T_{\nu n_1}^{\beta} \psi_{n_1} \bar{\psi}_{n_1} \dots \bar{\psi}_{n_1} + \gamma_\nu^{lk} (u_\alpha \beta_\nu^{\alpha\beta} u_\beta) \psi_k &= \eta_l; \\ \bar{\gamma}_\nu^{lk} \frac{\partial \bar{\psi}_k}{\partial x_\nu} - x' \bar{\psi}_l + T_{\beta n_1}^l S_{\alpha n_1}^{\beta} \dots T_{\nu n_1}^{\beta} S_{n_1 n_2}^{\nu} \psi_{n_1} \bar{\psi}_{n_2} \dots \psi_{n_1} - \bar{\gamma}_\nu^{lk} (u_\alpha \beta_\nu^{\alpha\beta} u_\beta) \bar{\psi}_k &= \bar{\eta}_l; \\ \beta_\nu^{lk} \frac{\partial u_k}{\partial x_\nu} + m u_l + N_{\alpha n_1}^l \dots N_{n_1 n_2}^{\alpha} u_{n_1} \dots u_{n_2} + (\bar{\psi}_l \psi_k) S_{kl}^i \bar{\psi}_i &= J_l, \end{aligned}$$

where β_l are matrices which satisfy the relations $\beta_l \beta_k \beta_l = \delta_{lk} \beta_l + \delta_{kl} \beta_l$; $N_{\alpha n}^l$ are Clebsch-Gordan coefficients. A similar operation enables us to obtain from equations (1) and (7) a system of interaction equations of higher spins and higher orders of nonlinearity. Thus we can reduce the entire set of interacting spinor, tensor and scalar fields to free fields described by nonlinear spinor equations. It is noteworthy that equations (1) and (7) are similar in form to the equations studied in papers [6] and [7].

5. We note that the nonlinear equations of Bose fields which were studied in the earlier paper can be written as

$$\left\{ -i N^{(a)} \frac{\partial}{\partial x_i} + J^{(a)} + \Gamma_l^{(B)} \frac{\delta}{\delta J_l} \right\} \Psi^{(B)} = 0. \quad (8)$$

Upon comparing (2) and (8) with the usual relativistically invariant equations we see that (2) and (8) differ from them by terms which are linear forms with respect to the second quantized momenta $\frac{\delta}{\delta \eta_l}$, $\frac{\delta}{\delta \bar{\eta}_l}$ and $\frac{\delta}{\delta J_l}$ [8]. These additional terms may be interpreted as mass operators, thus emphasizing the relation between non-local and nonlinear theorems.

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INTENSITY DISTRIBUTION IN THE OH-BAND OF THE SPECTRUM OF COMBINATION SCATTERING BY A CRYSTAL OF GYPSUM

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The problem of the shape of the OH-band in the spectra of combination scattering by crystals has thus far had very little experimental study. In our work [1] investigating the scattering spectrum of a gypsum crystal in the region of oscillations of the OH group we found a wide band with a complex structure. When the crystal is cooled to -196° the intensity of the band is decreased and a number of fairly sharp lines appear on a weak background. This structure of the OH-band in the combination scattering spectrum of gypsum is due to the linking of the intramolecular oscillations of the OH group and the intermolecular oscillations. The proposed interpretation of the structure of the OH-band is in good agreement with the theoretical ideas of B. I. Stepanov [2] and with the calculations of N. D. Sokolov [3].

In connection with our investigations it is of great interest to perform experiments with polarized light, thus making it possible to study the structure of the OH-band in greater detail. In the present work we performed such experiments to investigate the state of polarization of the OH-band in the scattering spectrum of gypsum ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$). We also investigated the effect of temperature on the shift of the fundamental frequencies of the OH-band for the purpose of explaining their character. In investigating the state of polarization of the OH-band of the scattering spectrum of a gypsum crystal we used the same apparatus that we had used previously [4].

Figure 1a shows the OH-band of the scattering spectrum of gypsum obtained with unpolarized light. The crystal in this case had its axis of symmetry (axis b) parallel to the beam of exciting light. As can be seen from Fig. 1a the OH-band in gypsum is very wide, attaining 650 cm^{-1} . In the middle of the OH-band on a comparatively weak background are observed two broad and intense lines with the frequencies $\nu_1 = 3405 \text{ cm}^{-1}$ and $\nu_2 = 3495 \text{ cm}^{-1}$. Symmetrically located on both sides of these lines there can be seen two maxima of considerably lower intensity. Figs. 1b and 1c show OH scattering bands associated with the two different components of the polarized scattered light. The spectrograms were obtained at a crystal temperature of 20° with a 15 hour exposure. An examination of these spectrograms shows that in the spectrum of the perpendicular component the OH-band extends from 3260 to 3720 cm^{-1} . The greatest intensity here compared with the 3405 cm^{-1} line is that of the 3495 cm^{-1} line, which lies in the middle of this polarized OH-band. The 3270 cm^{-1} line was also quite clearly discernible. On the other hand, in the spectrum of the parallel component, the middle of the band was occupied by the 3405 cm^{-1} line, which is considerably higher in intensity than the 3495 cm^{-1} line. The OH-band in this instance begins at approximately 3200 cm^{-1} and cuts off around 3630 cm^{-1} maximum.

Figure 2 shows the scattering spectrum of gypsum in the region of oscillations of the OH-group at crystal temperatures of 100° and -196° . Exposures were made in such a way that only the two fundamental intense lines appeared, without the accompanying weak bands. Comparison of the spectra shows very clearly that the frequencies ν_1 and ν_2 are shifted in different directions with change of temperature. When the crystal is heated the ν_1 line is shifted towards lower frequencies by $\Delta\nu_1 = 5 \text{ cm}^{-1}$, whereas the ν_2 line is displaced at the same time towards higher frequencies by $\Delta\nu_2 = 13 \text{ cm}^{-1}$.

We shall now consider the possible explanations of the observed phenomena. As is known from the theory of the hydrogen bond, the vibrational levels of the OH bond have a discrete structure due to the oscillations of the hydrogen bond. Therefore transitions between levels should result in the appearance, besides the fundamental strong line, of a number of additional weak lines distributed at identical distances from each other and symmetrically placed with respect to the single fundamental line. In our experiments with a gypsum crystal at low temperatures [1] we actually observed such a fine structure of the OH-band. However, instead of the single fundamental vibrational frequency of the OH group in the spectrum of gypsum, two strong lines are observed

with six weak lines located symmetrically around them. The present study of the scattering spectrum of gypsum with polarized light shows, as was indicated above, that the OH-band is really a superposition of two bands differing in the state of polarization and identical in appearance. Each of these bands consists of a strong line and a continuous background. The continuous background extends in both directions from the fundamental line for 230 cm^{-1} . The lines of the fine structure which lie on the continuous background are quite clear in the spectra of both components, whereas in the spectrum that is obtained with unpolarized light the structure of the band is seen much less clearly. In the latter case this is obviously due to the fact that the lines of the fine structure are covered by the continuous background when the two polarized bands are superimposed.

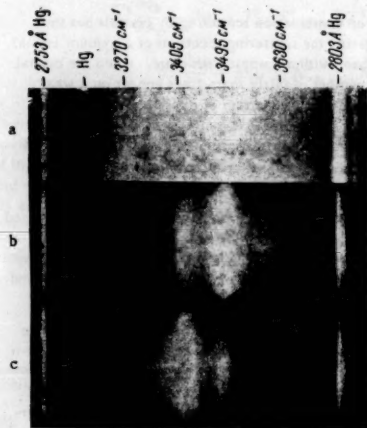


Fig. 1. a) OH-band of the combination scattering spectrum of a crystal of gypsum; b and c) components of the polarized scattering of light with electric vectors: b) perpendicular and c) parallel to the axis of the exciting light beam.

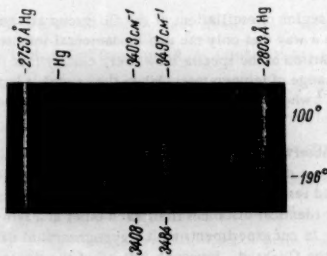


Fig. 2. Fundamental lines of the OH-band of the combination scattering spectrum of a gypsum crystal at temperatures of 100° and -196° .

Thus the results of our study of the polarization of the OH-band in a gypsum crystal show that the pattern of each component of the scattering spectrum of gypsum in the vibrational region of the OH-group finds a direct explanation in the theory of the hydrogen bond. However, the presence in the scattering spectrum of gypsum, in the region of OH-group valence vibrations, of two polarized bands, the fundamental lines of which differ strongly in their temperature dependence, seems to us to require special study.

The scattering spectrum of gypsum has been studied frequently. In the vibrational region of the OH-group the most thorough studies are those of Cabannes and Aynard who studied the dependence of the intensity of the lines $\nu_1 = 3405\text{ cm}^{-1}$ and $\nu_2 = 3495\text{ cm}^{-1}$ on the orientation of the crystal and also the polarization of these lines. On the basis of their experiments Cabannes and Aynard associate the ν_1 and ν_2 lines respectively with the symmetrical and antisymmetrical vibrations of the molecule of water of crystallization. It seems to us that this explanation of the frequencies ν_1 and ν_2 cannot be considered correct. If the ν_2 line is associated with the anti-symmetrical vibrations of the molecule of water of crystallization, it should be very weak in combination scattering. This is probably the reason why in the combination spectrum of water vapor a frequency corresponding to the anti-symmetrical vibrations of the water molecule has so far not been observed. In the spectrum of the combination scattering of water of crystallization one should therefore expect to observe frequencies corresponding primarily to the symmetrical vibrations of the water molecule.

Our studies of the shift of the fundamental frequencies of the OH-band with temperature show (see Fig. 2) that the lines ν_1 and ν_2 are shifted in different directions with a change of temperature. With increasing temperature the frequency ν_1 decreases but ν_2 increases. If we refer to studies of the dependence of the position of the OH-band on temperature we find, as is well known, that in the scattering spectra and infrared absorption spectra with increased temperature the frequency of the valence vibrations of the OH-group in crystals and liquids always shifts towards higher frequencies. This change of the frequency of the OH-group is explained by the theory of the hydrogen bond.

Therefore the anomalous temperature dependence of ν_1 leads to the assumption that the origin of ν_1 and ν_2 is not that which was suggested by Cabannes and Aynard.

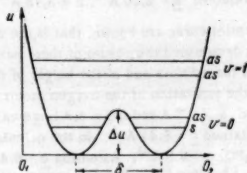


Fig. 3. Potential field along the direction of the hydrogen bond.

The appearance in the spectrum of gypsum of two fundamental frequencies in the region of valence vibrations of the OH-group we consider to be the result of a splitting of the symmetric frequency of the molecule of water of crystallization due to the tunnel effect.

The splitting of the vibrational levels caused by the tunnel effect has been most completely studied in the ammonia molecule and as we know [2] is associated with the characteristic form of the potential function, which has two minima. In some cases, in the presence of the hydrogen bond, the curve along which the proton moves also has two minima, corresponding to two equilibrium positions of the proton

(see Fig. 3). This was shown by Huggins [6] who investigated the hydrogen bond in water and in ice. Later the idea that the potential field along the direction of the hydrogen bond has the form shown in Fig. 3 was developed in the theory [7] of ferroelectric crystals containing the hydrogen bond. Recently the role of the hydrogen bond in the passage of a proton through the potential barrier has been investigated in theoretical work by N. D. Sokolov [8]. Thus at present it may be assumed that the potential curve along the hydrogen bond has two minima separated by a potential barrier. Therefore we consider it reasonable to assume that in substances where the hydrogen bond is present we may expect splitting of the vibrational levels of the OH-group and the appearance in the scattering spectrum and in the infrared absorption spectrum of two OH-bands.

Recently M. A. Kovner and V. N. Kapshtal [9], using a comparatively simple analytical expression for the potential curve, undertook a calculation of the potential barrier and of the separation of the minima of the potential curve for the ferroelectric crystal KH_2PO_4 . Their results are in good agreement with the data of U. Mezon [7] and of G. S. Landsberg and F. S. Barishanski [10]. However, the calculated value for the splitting of the OH-band is, as the authors indicate, too high.

We turn now to the conclusions which follow from the theory of the tunnel effect. As has been mentioned above, the existence of a potential curve with two minima causes a splitting of the vibrational levels of the molecule into two sublevels. The higher sublevel (as) always corresponds to the antisymmetrical wave function, and the lower level (s) corresponds to the symmetrical function. In the spectrum of combination scattering only the transitions (as) $\rightarrow$ (as) and (s) $\rightarrow$ (s) occur, thus giving two lines of the spectrum. The magnitude of such doublet splitting is directly linked with the probability of the passage of the particle from one potential well to the other; the stronger the splitting the greater the probability of finding the particle on different sides of the potential barrier.

Our experiments with a gypsum crystal are in agreement with these theoretical deductions. In the scattering spectrum of gypsum we have found that the OH-band is split into two differently polarized bands whose separation depends on temperature. With increased temperature the splitting is enhanced as both components of the OH-band are shifted in different spectral directions (Fig. 2). It seems to us that an enlargement of the OH-band splitting with temperature must be associated with the fact that the probability of a passage of the proton from one potential well to the other increases with rising temperature, so that we are justified in expecting greater splitting of the OH-band at high temperatures. This interpretation of the temperature dependence of the splitting of the OH-band in gypsum crystals agrees with the results of our investigations.

Confirmation of the correctness of the above interpretation of the fundamental frequencies of the OH-group in gypsum crystals is also given by an evaluation of the dissociation energy of the hydrogen bond in gypsum.

According to the calculations of N. D. Sokolov [8] the relative shift of the OH-band due to the hydrogen bond can be given in the following form:

$$\frac{\Delta \nu}{\nu_0} = \frac{\epsilon_0}{K}$$

where $\kappa = \frac{4a^2D}{3ab-b^2+4c-2c^2}$; ϵ_0 is the energy of the hydrogen bond; Δ is the energy of an isolated OH-bond, equal to 110 large calories/mole; a , b and c are theoretical parameters: $a = 2.30 \text{ \AA}^{-1}$, $b = 3.78 \text{ \AA}^{-1}$.

The parameter c can be determined if the parameters of the potential curve are known, that is, the distance δ between the minima and ΔU the height of the potential barrier. We determined the values of these parameters from Mezon's [7] curve which gives the dependence of the separation of the minima and of the height of the potential barrier on the length of the hydrogen bond. Considering that the separation of the oxygen atoms in a gypsum crystal according to Wooster's data [11] is 2.70 or 2.71 $\text{\AA}$ we find $\delta = 0.7 \text{ \AA}$ and $\Delta U = 4.4$ large calories/mole. Using these values of the parameters of the potential curve we obtained $c = 6.44 \text{ \AA}^{-1}$. In the calculations of M. A. Kovner and V. N. Kapshtal [9] for the ferroelectric crystal KH_2PO_4 , $c = 6.3 \text{ \AA}^{-1}$. Assuming $c = 6.44 \text{ \AA}^{-1}$ and using our data on the position of the band in the scattering spectrum of gypsum ($\Delta\nu/\nu_0 = 0.055$) we found that the dissociation energy of the hydrogen bond in a gypsum crystal is $\epsilon_0 = 4.8$ large cal./mole. The energy of the hydrogen bond found in this way is undoubtedly close to its actual value in a gypsum crystal.

In conclusion, consider it a pleasant duty to thank corresponding member of the Academy of Sciences of the U.S.S.R. E. F. Gross for his interest and for a discussion of the results.

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AN ELECTRON MICROSCOPIC STUDY OF THIN FILMS OF ANTIMONY BY THE DARK FIELD METHOD

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(Presented by Academician N. V. Belov July 16, 1955)

The use of the dark field method in electron microscopy enables us just as in the case of the ordinary light microscope to increase image contrast. As the contrast is increased it is possible from the character of the observed image to determine the crystalline part of the object. As a result of the parallel use of dark field and light field electron microscope and electron diffraction methods of investigation it has been shown that intense and sharp images of the details of structure in a dark field picture are given by electrons reflected from the crystallographic planes [1]. Depending on the nature of the object the image is formed predominantly either by diffusely scattered electrons (for objects with weakly ordered structure) or by electrons reflected from crystallographic planes (for crystals).

For diffusely scattered objects the quality of the dark field image is inferior to the light field image because of the imperfection of the objective and depends on the means used to obtain the dark field image. If the dark field image is obtained by inclining the condenser system, the objective works under identical conditions for the formation of dark field and light field images and astigmatism is absent. But the quality of the dark field and light field images will be identical only for crystalline objects. For weakly ordered or irregular structures the quality of the dark field and light field images will differ. This latter circumstance is explained by the fact that the intensity distribution in a light field image is determined mainly by the number of electrons proceeding at an angle determined by the aperture of the condenser. The number of electrons scattered at an angle determined by the aperture of the objective is in this case incomparably smaller than the number of electrons proceeding at an angle given by the aperture of the condenser, thus producing only a small decrease of image contrast. In the case of the dark field the intensity distribution in the image is determined by the electrons proceeding at an angle determined by the aperture of the objective which is one order larger than the aperture of the condenser. The scattering circle due to spherical aberration is in this case 10^3 larger. In proportion to this quantity the quality of the dark field image is reduced for diffusely scattering objects. For crystals from which the interference pulses of Wulf-Bragg reflection pass through a diaphragm, the scattering circle both in a dark field and in a light field image is determined by electrons proceeding at an angle given by the aperture of the condenser.

When the dark field image is obtained by adjusting the diaphragm the scattering circle due to spherical aberration also depends on the angle determined by the aperture of the objective for weakly ordered objects or of the condenser for crystalline objects; in addition it depends on the angle which is given by the displacement of the diaphragm with respect to the optical axis. As shown by calculation, the error due to spherical aberration when a dark field image is obtained by displacement of the objective diaphragm exceeds by 4 times the error for a dark field image obtained by inclination of the condenser system (the scattering circle in this case is transformed into an ellipse).

Thus, all other conditions being equal, the quality and intensity of the dark field image of crystalline objects are considerably higher than the quality of the image of objects with weakly ordered structure. In practice intense sharply defined structural details can be observed only in substances with a high degree of orientation. This characteristic of the dark field image enables us to determine to which class of substances, crystalline or amorphous, a given object belongs. Great interest attaches to the application of the dark field method to the investigation of very thin films of substances whose diffraction picture consists of broad diffuse rings.

Thus, for example, the problem of the structure of very thin films of antimony has long attracted the

attention of researchers [2]. After slow evaporation of antimony on to a neutral backing to a thickness of 100 Å, a diffraction pattern appears with broad diffuse rings which cannot be interpreted according to the structure of metallic antimony. Some authors [2, 3] consider this a proof of the existence of amorphous antimony. However, it is known that an increase in the diffuseness of the electronogram and a distortion of the relative intensity of the interference pulses can be the result of a decrease in the size of the crystals as well as of a transition to the amorphous state.

The characteristics of the dark field method enable us to determine the character of the structure of such a film of antimony. For this purpose a film of antimony about $0.5 \mu\text{g}/\text{cm}^2$ thick, evaporated on to a neutral collodion backing was studied with a UEM-100 versatile electron microscope which enabled us to observe the image on a dark and light field and at the same time obtain an electron diffraction pattern of that portion of the object. The accelerating voltage was 80 kv. The dark field image was obtained by shifting the diaphragm in the horizontal plane.

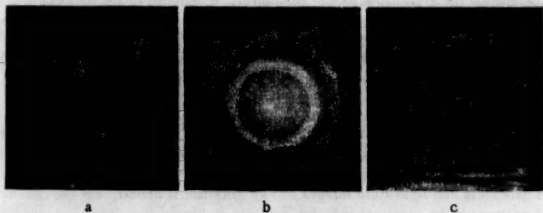


Figure 1. Layer of antimony $0.5 \mu\text{g}/\text{cm}^2$ thick on a collodion film. a) In a light field; c) in a dark field, $\times 12,000$; b) electron diffraction pattern. Accelerating voltage 80 kv.



Figure 2. Layer of antimony $0.5 \mu\text{g}/\text{cm}^2$ thick after prolonged irradiation with electrons. a) In a light field; c) in a dark field, $\times 12,000$; b) electronogram. Accelerating voltage 80 kv.

From observation of this film of antimony in a light field it appeared that it consists of individual particles of antimony measuring up to 150 Å (see Fig. 1a). The intrinsic structure of particles of antimony cannot be determined by light field microphotography. The diffraction of electrons by such a film gives a characteristic pattern — in the electronogram 4 diffuse rings are seen which correspond to $d = 3.0$ Å — strong; 1.94 Å — medium; 1.47 Å — weak; 1.26 Å — weak (the remaining 3 rings were cut from the field of view by the diaphragm). The average size of the particles as estimated from the electronogram is 70 Å. At the same time observation of this portion of the object in a dark field may reveal individual very distinct bright spots which contrast with the weak gray background (see Fig. 1c). The bright spots in the dark field microphotograph indicate a sharp reinforcement of individual interference points, which is evidence of the existence of crystals in this antimony

film, since only in the case of a crystalline structure of the object can a high intensity of scattered electrons be obtained in a small angle with the formation of a clear interference maximum. In the dark field microphotograph, the minimum size of the crystals in the film can be determined, amounting to 30-35 Å.

The thermal activity of the electrons in the film due to an increase in the intensity of irradiation induced the growth of the crystals in the antimony film, which is especially clear in the dark field microphotograph (Fig. 2 c); the nature of the bright spots has not changed, but the intensity of the pulses increased, as is quite natural after an increase in the size of the crystals. An electronogram consisting of a large number of thin rings in this case gives a pattern that is typical for metallic antimony (Fig. 2 b). This fact may be considered a proof that the bright spots in a dark field microphotograph of this antimony film are caused by antimony crystals and not by a foreign substance.

The presence of sharp bright points in a dark field image from a thin antimony film may be considered proof that it consists of small crystals rather than of amorphous particles, even when the diffraction pattern consists of broad diffuse rings which cannot be interpreted in accordance with the structure of metallic antimony. Apparently in the case of a very thin film of antimony there is a distortion of the crystalline structure which induces a displacement of lines in the electronogram that is accompanied by an increase in diffuseness of the electronogram due to decreased crystal size.

These results showed that use of the dark field method of studying thin films of antimony (as a supplement to the light field electron microscope and electron diffraction methods) enables us to obtain more accurate information concerning the structure of a substance than can be obtained by the exclusive use of the electronographic method alone.

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THE STRUCTURE OF SIMPLE LIQUIDS

A. E. Glauberman and V. P. Tsvetkov

(Presented by Academician N. N. Bogoliubov July 21, 1955)

Recently considerable progress has been made in the theory of classical systems of interacting particles, especially in connection with the rigorous methods developed by N. N. Bogoliubov [1].

However, Bogoliubov's equations for the configurational distribution functions of groups of particles in the case of a liquid do not contain a small parameter, and for their approximate solution we are obliged to introduce a "superpositional" approximation which in the equation for a binary function has the form

$$F_{abc} = \frac{F_{ab} F_{ac} F_{bc}}{F_a F_b F_c} \quad (1)$$

The adequacy of this approximation can be determined only by a detailed calculation of the errors which it introduces.

For a solid sphere model, an expansion in powers of the density was used to find the difference between the rigorous and approximate solutions in the term of the series which is proportional to the square of the density [2]. For Coulomb forces acting between the particles an evaluation shows a marked difference in the term containing the second power of the small parameter [3]. Calculations performed by one of the authors of the present paper and his collaborators showed that for a system of particles interacting according to an arbitrary form of central short-range forces a difference begins to appear with the term that is proportional to the square of the density. A calculation of higher approximations enabled us to estimate this difference, with the result that the error appearing in the second approximation (i.e., in the term which is proportional to the square of the density) is greater in order of magnitude than the third approximation in an exact solution. Similar quantitative results are found in [3] in connection with the general form found for the terms of the series under investigation.

The above results indicate that for such large densities as exist in a liquid the superpositional approximation is, generally speaking, unsuitable. In each case, for a study of the liquid structure it is necessary, in addition to theories based on application of the superpositional approximation, to develop the so-called model theories.

A theory of a similar order for simple liquids of quasicrystalline structure, which was developed in [4, 5], was improved by one of us [6]: a radial distribution function was found in the form

$$G(r) = \sum_s \frac{n_s}{V \pi (\alpha + 4 D r_s)} \exp \{-(r-r_s)^2 / \alpha + 4 D r_s\}, \quad (2)$$

where n_s is the number of atoms at a distance r_s from the atom chosen to be at the center of the crystal lattice; $\alpha = kT/f$, where f is the quasielastic binding factor; D is the so-called coefficient of structural diffusion, i.e., the coefficient characterizing the statistical scatter in the equilibrium positions of the atoms of the liquid compared with their corresponding positions in a crystal lattice, which appears when long-range order is eliminated.

An important problem of this model theory is the explanation of the nature of the temperature dependence of the coefficient of structural diffusion D . This problem can be solved by the method given in [6].

Using the familiar formula

$$4\pi r^2 \rho(r) = 4\pi r^2 \rho_0(r) + \frac{2r}{\pi} \int_0^\infty s i(s) \sin(sr) ds, \quad (3)$$

where $\rho(r)$ is the radial atomic density and ρ_0 is the density of uniform distribution; $i(s)$ is the intensity of x-ray scattering in the liquid as a function of the scattering angle ($s = 4\pi \sin \theta / \lambda$), from the experimentally determined function $i(s)$ we obtain in accordance with (3) the atomic distribution function. With the experimental distribution function obtained in this way we compare the distribution function constructed by using Equation (2).

By mapping the experimental curves of $\rho(r)$ at various temperatures and matching them to theoretical curves for corresponding temperatures it is possible to determine the temperature dependence of the coefficient of structural diffusion D .

In connection with the fact that Equation (2) is applicable generally to a temperature region which is not too close to the temperature of crystallization [6], it is necessary to carry out a "reduction" of the theoretical curves as follows. Assuming $D = 0$ at the crystallization point, in Equation (2) the magnitude of f is determined such that the theoretical distribution curve agrees as well as possible with the experimental curve at the temperature closest to the crystallization point.

In the present paper the above method was used for liquid tin. As initial crystal lattices to be subjected to structural "erosion" we considered the lattices of white tin, gray tin and the close-packed hexagonal lattice. The best agreement of the experimental and theoretical curves is obtained for the washing-out of the close-packed hexagonal lattice (lattices differing from the above three types give entirely unsatisfactory results).

Experimental curves of the atomic distribution were obtained for the temperatures: 237°, 252°, 269°, 284°, 301°, 315°, 329°, 345°, 361°, 375°, 392°, and 407°. For tin the "reduced" value of the quasielastic binding factor f is $0.7 \cdot 10^4$ dynes/cm.

Figure 1. Atomic distribution functions of liquid tin at different temperatures. A) Experimental curves; B) theoretical; C) curves of the uniform distribution $2\pi r^2 \rho_0$; D) at 237°; 1) 269°; 2) 315°; 3) 435°; 4) 407°. a_1, a_2, a_3, a_4 are zero points of the respective curves.

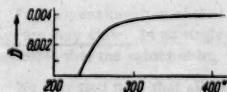


Figure 2. Temperature dependence of the structural diffusion coefficient D .

Roentgenograms of liquid tin were obtained by using radiation from copper which had been monochromatized by the method of differential filters. This method enabled us to obtain photographs in short periods without appreciable oxidation of the specimen. A special vacuum chamber was used with a movable film holder in order to obtain two photographs on one film. The diameter of the cylindrical film was 57.3 mm. After microphotometric study and corrections for polarization and absorption intensity curves were constructed in electron units. From these curves the atomic distribution function was constructed.

Figure 1 shows part of the atomic distribution curves obtained in experiment (A) and the theoretical curves constructed from Equation (2) with an initial tightly packed hexagonal lattice and an optimal choice of values of D for each value of the temperature (B), thus giving very satisfactory agreement of the theoretical and experimental curves.

Figure 2 shows the temperature dependence of the structural diffusion coefficient which was obtained in this way for the entire temperature range that was investigated.

Detailed experimental investigations of the atomic distribution functions in liquids at different temperatures, such as those described in the present article, should accompany the large amount of data accumulated by V. I. Danilov and his pupils; and, in addition, the binary model distribution functions determined by Equation (2) can then be used to describe the properties of liquids for practical purposes.

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EQUALLY ALLOWED BETA TRANSITIONS

B. Z. Dzhelepov, Corr. Memb. Acad. Sci. USSR, and L. K. Peker

1. It is well-known that the probability of a β -transition depends on the change of the total moment of the system ΔI , on the change in parity of the wave function of the system Δd , at least in light nuclei, and on the change of the orbital moment of the system ΔL . If the changes of these quantities are within the limits dictated by the selection rules the β -transition is allowed; if the changes ΔI , Δd , ΔL lie outside these limits the β -transition is forbidden.

Two questions arise: 1) Can only the above factors forbid β -transitions, and 2) to what extent can the products τf differ in transitions in which ΔI , Δd and ΔL are known to be identical?

In order to find answers to these questions we obtained values of $\log \tau f$ in β -transitions in which ΔI , Δd and ΔL should be identical, that is, for: a) competing β^+ - β^- decays in nuclei with even A and odd Z (see Figure 1a); b) successive β -decays in isobaric triplets in which the central isobar has even A and odd Z (see Figure 1b). In both cases the outer isobars are nuclei with even A and Z in which, according to current ideas, the ground states have total and orbital moments equal to zero and even wave functions.

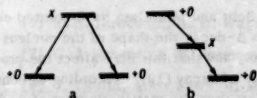


Figure 1.

X) Ground state of an even-odd nucleus;
+0) ground state of an even-even nucleus.

We limit ourselves to a consideration of only the transitions between ground states of nuclei. It is obvious that for any values of I , d and L in the X nuclei (Figure 1) both β -transitions in which these nuclei take part have identical ΔI , Δd and ΔL .

A comparison of the values of $\log \tau f$ for these transitions should give us information as to whether the probability of a β -transition depends on any factors besides ΔI , Δd and ΔL .

In some particular cases such a comparison has already been made [1]. However, by now the experimental data have been considerably extended: we have the required information for 10 decays of type a) and 11 decays of type b). In Table 1 all these decays are listed and the values of $\log \tau f$ are given.

From an examination of the table we can draw the conclusion that the products τf in each pair of decays are very close. In no single case does $\log \tau f$ differ by more than 1.2; this difference is small if it is remembered that the values of $\log \tau f$ for different β -emitters are spread over a range from 3 to 18.

We may thus state that all 21 pairs of β -decays have an identical order of forbiddenness.

2. From the comparison of $\log \tau f$ for the paired β -decays the following rule may be formulated:

If one of two extreme nuclei in Table 1 has a filled proton or neutron shell, $\log \tau f$ for a transition associated with this nuclei is smaller than for another transition.

The corresponding cases are given in the first nine lines of Table 1. The above rule is observed in 8 cases and not observed in one case (No. 8).

Case No. 10, in which both extreme nuclei have filled shells, is one to which the rule cannot be applied. If the different values of $\log \tau f$ for $\text{Ne}^{18} \rightarrow \text{F}^{18}$ and $\text{F}^{18} \rightarrow \text{O}^{18}$ decays are confirmed this will indicate that besides the degree of filling of the shells there are other factors affecting the magnitude of $\log \tau f$.

Table 1

 β -Transitions of the Types $O^+ \rightarrow X \rightarrow O^+$ and $O^+ \rightarrow X \rightarrow O^+$

No.	β -Transitions and $\log \tau_f$	Literature
1	$^{64}_{28}\text{Ni} \xrightarrow{4.96 \pm 0.02} ^{64}_{20}\text{Cu} \xrightarrow{5.29 \pm 0.01} ^{64}_{31}\text{Zn}$	[2]
2	$^{112}_{48}\text{Cd} \xrightarrow{4.6 \pm 0.3} ^{112}_{49}\text{In} \xrightarrow{4.06 \pm 0.06} ^{112}_{50}\text{Sn}$	[2, 3]
3	$^{114}_{48}\text{Cd} \xrightarrow{5.0 \pm 0.3} ^{114}_{49}\text{In} \xrightarrow{4.42 \pm 0.01} ^{114}_{50}\text{Sn}$	[4]
4	$^{204}_{80}\text{Hg} \xrightarrow{9.9 \pm 0.2} ^{204}_{81}\text{Tl} \xrightarrow{2.64 \pm 0.06} ^{204}_{82}\text{Pb}$	[5]
5	$^{88}_{36}\text{Kr} \xrightarrow{7.75 \pm 0.29} ^{88}_{37}\text{Rb} \xrightarrow{7.33 \pm 0.06} ^{88}_{38}\text{Sr}$	[2, 8]
6	$^{90}_{38}\text{Sr} \xrightarrow{9.23 \pm 0.04} ^{90}_{39}\text{Y} \xrightarrow{8.01 \pm 0.06} ^{90}_{40}\text{Zr}$	[2]
7	$^{212}_{82}\text{Pb} \xrightarrow{6.81 \pm 0.04} ^{212}_{83}\text{Bi} \xrightarrow{7.23 \pm 0.05} ^{212}_{84}\text{Po}$	[2]
8	$^{82}_{28}\text{Ni} \xrightarrow{5.1 \pm 0.2} ^{82}_{29}\text{Cu} \xrightarrow{4.96 \pm 0.06} ^{82}_{30}\text{Zn}$	[5]
9	$^{140}_{58}\text{Ce} \xrightarrow{4.7 \pm 0.2} ^{140}_{59}\text{Pr} \xrightarrow{4.7 \pm 0.2} ^{140}_{60}\text{Nd}$	[7]
10	$^{18}_{8}\text{O} \xrightarrow{3.43 \pm 0.07} ^{18}_{9}\text{F} \xrightarrow{2.9 \pm 0.2} ^{18}_{10}\text{Ne}$	[2]
11	$^{32}_{14}\text{Si} \xrightarrow{6.7 \pm 0.3} ^{32}_{15}\text{P} \xrightarrow{7.92 \pm 0.03} ^{32}_{16}\text{S}$	[2]
12	$^{74}_{32}\text{Ge} \xrightarrow{8.57 \pm 0.03} ^{74}_{33}\text{As} \xrightarrow{8.32 \pm 0.03} ^{74}_{34}\text{Se}$	[2]
13	$^{86}_{34}\text{Se} \xrightarrow{4.23 \pm 0.06} ^{86}_{35}\text{Br} \xrightarrow{5.44 \pm 0.03} ^{86}_{36}\text{Kr}$	[2]
14	$^{102}_{44}\text{Ru} \xrightarrow{9.1 \pm 0.1} ^{102}_{45}\text{Rh} \xrightarrow{9.8 \pm 0.1} ^{102}_{46}\text{Pd}$	[2]
15	$^{106}_{44}\text{Ru} \xrightarrow{4.3 \pm 0.1} ^{106}_{45}\text{Rh} \xrightarrow{5.23 \pm 0.02} ^{106}_{46}\text{Pd}$	[2]
16	$^{108}_{46}\text{Pd} \xrightarrow{4.8 \pm 0.3} ^{108}_{47}\text{Ag} \xrightarrow{4.63 \pm 0.04} ^{108}_{48}\text{Cd}$	[2]
17	$^{126}_{52}\text{Te} \xrightarrow{8.5 \pm 0.1} ^{126}_{53}\text{I} \xrightarrow{8.92 \pm 0.03} ^{126}_{54}\text{Xe}$	[2]
18	$^{130}_{54}\text{Xe} \xrightarrow{5.08 \pm 0.05} ^{130}_{55}\text{Cs} \xrightarrow{4.96 \pm 0.04} ^{130}_{56}\text{Ba}$	[2]
19	$^{144}_{58}\text{Ce} \xrightarrow{7.43 \pm 0.04} ^{144}_{59}\text{Pr} \xrightarrow{6.53 \pm 0.02} ^{144}_{60}\text{Nd}$	[2]
20	$^{234}_{90}\text{Th} \xrightarrow{6.44 \pm 0.05} ^{234}_{91}\text{Pa} \xrightarrow{5.52 \pm 0.04} ^{234}_{92}\text{U}$	[9]
21	$^{240}_{82}\text{U} \xrightarrow{5.6 \pm 0.3} ^{240}_{83}\text{Np} \xrightarrow{6.53 \pm 0.03} ^{240}_{84}\text{Pu}$	[2]

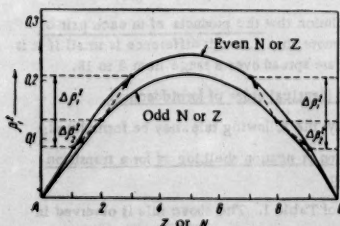


Figure 2. The ordinate axis shows values of the deformation coefficient β^2 (in arbitrary units). The axis of abscissas gives the number of nucleons in a filled level. The point A corresponds to the start of filling of a level and the point B to completion of filling. A level with $j = 9/2$ has been chosen arbitrarily.

In cases Nos. 11-21 the shells are not filled. However, it may be noted that $\log \tau_f$ is smaller for transitions involving nuclei which have almost filled shells (Nos. 16, 17 and 19) or those involving nuclei in which there are 14, 34, 56 and 92 protons or 58 and 148 neutrons (the possible subshells).

3. Researches of the last few years show that the surface of the nuclei may be deformed because of nucleons outside a filled shell.

In first approximation the equation of this deformed surface deformation can be given as

$$R = R_0 [1 + \beta Y_{2,0}(\cos \theta)],$$

where R_0 is the radius of the equivalent spherical nucleus; $Y_{2,0}(\cos \theta)$ is the normalized spherical function, and the coefficient β is a measure of the degree of surface deformation.

Bohr and Mottelson have pointed out that in β -decay the shape of the nucleus changes, and that this may affect the probability of decay [10]. According to Bohr and Mottelson the matrix element of β -decay allowing for a change of shape of the nucleus can be given as

$$M^2 = M^2_{1\text{-particle}} F.$$

The factor F depends on the deformation parameter β_1 in the initial and final states of the nucleus and describes the dependence of the β -decay probability on the change of nuclear shape. The factor is equal to 1 if there is no change of shape and is < 1 if the shape changes.

The above formula was obtained under the assumption that in β -decay the state of only one nucleon changes. In reality β -decay is accompanied not only by a change of shape but by some change of the internal structure of the nucleus [11]. The effect of a change of nuclear structure is included in the matrix element by replacing $M^2_{1\text{-particle}}$ with an arbitrary M_0 ; by F we shall as previously mean a factor depending only on the shape of the nucleus. We shall show that calculation of the factor F may be used to explain the basis for the rule in Section 2.

Let us consider the dependence of the deformation coefficient on the number of nucleons in a filled shell.

Wilets, Hill and Ford [12] and others [13, 14] have shown that in order to explain the uneven curve of isotopic shifts in a sequence of even and odd isotopes it must be assumed that the deformation coefficients of isotopes with odd A are somewhat smaller than the deformation coefficients of isotopes with even A .

We shall make use of this hypothesis. Figure 2 shows the dependence of β_1^2 on the number of protons in a filled shell for nuclei with even and odd N (number of neutrons). It is easily seen that the magnitude of β^2 for a β -transition which brings a nucleus closer to having a filled shell changes less ($\Delta\beta^2$ less) than for a transition which takes a nucleus farther from such a state. To larger $\Delta\beta$ there correspond a larger change of the shape of the nucleus, smaller F and, consequently, a larger value of $\log \tau f^*$. This is in agreement with the rule in Section 2.

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* Bohr and Mottelson [10, 15] in their calculations of the deformation coefficient from excitation energy of rotational levels obtained somewhat larger values of β for odd nuclei than for nuclei with even A , which disagrees with the data in [12-14]. However, as was shown in [11], the energy levels cannot give correct values of the moments of inertia and, consequently, of the nuclear deformation coefficients.

RELATIVE INTENSITY OF THE FINE STRUCTURE COMPONENTS OF He II λ 4686 A

P. S. Kireev

(Presented by Academician L. A. Artsimovich July 20, 1955)

A considerable number of papers have appeared recently on the fine structure of hydrogen and singly-ionized helium lines for the purpose of testing the theory of the shift of the $ns_{1/2}$ term with respect to the $np_{1/2}$ term, which terms coincide according to Dirac's theory. An indication of their separation was obtained from spectroscopic data. The final proof resulted from the experiments of Lamb and Retherford.

However, the majority of authors who investigate the fine structure confine themselves to a measurement of the separation of the components. The intensity of the fine structure components of the He II lines has not been measured although Dirac's radiation theory gives exact values for them. It is of great interest to obtain an experimental verification of Dirac's theory for the intensities. Paschen [1], Leo [2] and Series [3] have compared experiment with theory qualitatively.

The present paper presents quantitative measurements of the relative intensity of the fine structure components of He II λ 4686 A under different conditions of excitation.

As a premonochromator we used a triple prism ISP-51 spectrograph with $anf = 270$ mm camera. To resolve the fine structure a Fabry-Perot interferometer was used with a 50 mm diameter of the working surfaces of the mirrors and reflection coefficient of 88% around 4700 A. The interferometer was placed in front of the slit of the spectrograph; the second-order image was taken through an additional slit. The optical scheme enabled us to utilize the radiation from the source completely with uniform illumination of the spectrograph slit. The light source was a discharge tube with a hollow cathode, of hollow diameter 10 mm and length 100 mm. The anode and the cathode were made of aluminum. In order to reduce the Doppler breadth of the lines, the source was cooled with liquid nitrogen. The current was varied from 30 to 500 ma, with the time of exposure varying from 30-60 minutes to 2-5 minutes.

Table 1

Designation	Transition	Theoretical intensity	Designation	Transition	Theoretical intensity
α	$3d_{5/2} - 4f_{7/2}$	0.3835	ϵ	$3p_{1/2} - 4d_{5/2}$	0.0821
β	$3d_{5/2} - 4f_{5/2}$	0.01917	μ	$3s_{1/2} - 4p_{3/2}$	0.04245
γ	$3d_{5/2} - 4f_{3/2}$	0.4150	ξ	$3d_{3/2} - 4p_{3/2}$	0.00410
δ	$3p_{3/2} - 4d_{5/2}$		χ	$3d_{3/2} - 4p_{1/2}$	0.00274
η	$3p_{1/2} - 4d_{5/2}$	0.0167	φ	$3p_{3/2} - 4s_{1/2}$	0.00821
ν	$3d_{3/2} - 4p_{3/2}$		ψ	$3s_{1/2} - 4p_{1/2}$	0.00218
			ω	$3p_{1/2} - 4s_{1/2}$	0.00410

The spectrograms were studied with a microphotometer. A comparator scale was recorded on each film with corresponding time of exposure. The characteristic curve of the film $S(\log I)$ was transformed into a curve $S(I)$ by which the blackening of the spectrogram was converted into intensity. The intensity curve constructed in this way for lines with complex structure gives at each point the total intensity of the interference picture of all components. To separate the interference curves of the components or of the group of unresolved components of the complex, the overall contour was resolved into components graphically. Since

Table 2

 $\tau = 2.5 \text{ mm}$

	Complex									
	α	I_{α}/I_{γ}	γ	I_{ψ}/I_{γ}	ψ	$I_{\psi'}/I_{\omega}$	ω	$I_{\psi'}/I_{\gamma}$	φ	
	Components									
	$\alpha, \beta, \nu, \eta, \epsilon, \mu$		γ, δ		ψ		ω		φ, χ, ξ	
I_{theor}	0.5439	1.31	0.415	0.0528	0.0218	5.33	0.0041	0.0369	0.01505	
I_s	0.5376	1.29	0.416	0.0470	0.0196	3.55	0.0055	0.054	0.0226	

Table 3

 $\tau = 2.0 \text{ mm}$

	Complex					
	α	$\beta, \nu, \eta, \epsilon, \mu$	γ	δ	ψ	ω
	I_{α}/I_{γ}	$I_{\beta, \nu, \eta, \epsilon, \mu}/I_{\gamma}$	I_{δ}/I_{γ}	I_{ψ}/I_{γ}	I_{ω}/I_{γ}	I_{φ}/I_{γ}
I_{theor}	0.4193	0.415	0.0218	0.0041	0.1395	
I_s	0.4040	0.417	0.0180	0.0078	0.1520	

it was impossible to resolve the direct intensity curve due to the nonlinear dispersion of the interferometer, the intensity curve was converted for linear dispersion.

The intensities of the components and complexes were obtained by measurements of the area bounded by the contour of a given complex and by measurements of the maxima. The intensity measured according to the area of a complex corresponds to the sum of the intensities of the components of the complex.

Table 1 gives the spectral symbols of transitions for components of the fine structure, their designations and theoretical values of the intensities of the components relative to the total intensity of the line.

Tables 2 and 3 give results of measurements carried out with 2.5 and 2.0 mm thick interferometers at a helium pressure of 0.5 mm Hg.

From Table 2 it can be seen that the relative intensity of complexes α and γ agrees well with the theoretical values, from which we may conclude that the theory gives correct values for the intensity of the $\alpha, \beta, \eta, \nu, \epsilon, \mu, \gamma, \delta$ components if we consider the intensity of these complexes as expressed by the total intensity of the line. Good agreement is shown in Table 3 for these same components. The intensity of the components ψ agrees with the theoretical value, but since the intensity of the component is small, the accuracy of the measurement for this component is less than for more intense components. Considerable deviations are observed for the component ω and the complex φ : their intensity exceeds the theoretical values. The strength of the ω component was observed by Mack and Austern [4], the ω component and the φ complex by Series [3]. An anomalous ratio of the intensities $I_{\alpha} : I_{\gamma}$ was observed by Leo [2]. In the present work, an anomalous ratio $I_{\alpha} : I_{\gamma}$ was observed with a helium pressure in the light source of the order of 4 mm Hg. The ratio of the maxima $I_{\alpha} : I_{\gamma}$ is 1.130, which exceeds this ratio at a pressure of 0.5 mm Hg by 25%. The intensity of the remaining components differs insignificantly from their values at a pressure of 0.5 mm Hg.

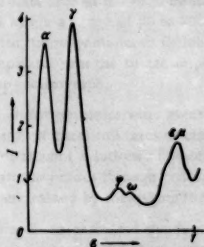


Figure 1. Intensity of He II λ 4686 Å. 2 mm etalon. Helium pressure 0.5 mm Hg.

An explanation of the increased values of the intensity must be sought in the dependence of the intensity on the conditions of excitation, which is not taken into account by the theory of spontaneous emission. It would

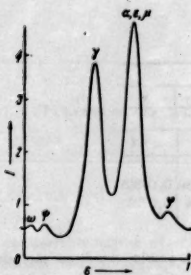


Figure 2. Intensity of He II λ 4686 Å. 2.5 mm etalon. Helium pressure 0.5 mm Hg.

gives the intensities in arbitrary coordinates. The horizontal axis gives the order calculated from a certain point of the interference pattern of the lines with linear dispersion. Both figures show good agreement of the components ψ and ω . The results of measurements of the separation of the components ψ and γ , and ψ and ω gives for shifts of the terms the values: $\delta E_{35} = 129.1 \cdot 10^{-3} \text{ cm}^{-1}$, $\delta E_{45} = 57.9 \cdot 10^{-3} \text{ cm}^{-1}$, which is in good agreement with the data of Bethe and with the experimental results of several authors (according to Bethe $\delta E_{35} = 130 \cdot 10^{-3} \text{ cm}^{-1}$, $\delta E_{45} = 54 \cdot 10^{-3} \text{ cm}^{-1}$).

In conclusion I wish to express my sincere thanks to F. A. Korolev for directing this research.

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PHOTOFISSION CROSS SECTIONS OF URANIUM AND THORIUM

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(Presented by Academician D. V. Skobel'tsyn November 21, 1955)

In 1949 and 1950 we studied the dependence of the uranium and thorium fission cross sections on the energy of gamma radiation [1]. There is a considerable spread of values of the photofission cross sections of heavy nuclei in the literature [2-4].

Our experiments were carried out with bremsstrahlung from the synchrotron of the P. N. Lebedev Physical Institute of the Academy of Sciences of the U. S. S. R. in the maximum energy range of the bremsstrahlung spectrum from 8 to 24 Mev. For the measurements we used a differential ionization pulse chamber filled with air. The electrodes of the chamber were three parallel disks. A layer of the investigated material was deposited on the inner side of one of the outer electrodes. The middle disk served as a collecting electrode and was connected to the control grid of the first amplifier tube.

Pulses of fission fragments originating in one half of the chamber were measured against a background of pulses from the synchrotron gamma radiation. By changing the potential on the high-voltage electrodes of the chamber these γ -pulses which arise in both halves of the chamber and flow to the grid of the first tube with different signs can be compensated up to the noise level of the amplifier.

Fragment pulses were recorded with apparatus distinguished by simplicity of adjustment and control and showing little sensitivity to the different kinds of interference which appear in work with a synchrotron.

The linear preamplifier was installed in the same housing as the ionization chamber and was connected to the main amplifier by a 5 m cable.

The nonuniformity of the frequency characteristic of the amplifier from 500 cycles to 3.5 megacycles did not exceed 3 db. From the main amplifier the pulses were fed to a pulse broadening circuit and to a discriminator with variable threshold, and then to a time selection circuit which selected the operating time interval during which the final stages of the apparatus received and recorded pulses. This section operated from the later side of the high frequency synchrotron pulse. The start and duration of the operating interval changed within a range of 20 to 200 μ sec, which considerably reduced the probability of recording pulses from noise, from spontaneous fission and from random alpha particles. The stability of amplification was tested separately for the linear amplification sections and for the output stages. Pulses were recorded on film by a loop oscillograph.

Photofission yields were measured on samples with 1.76 mg/cm^2 of uranium and 0.8 mg/cm^2 of thorium. Natural mixtures of uranium and thorium were used. The samples were prepared by multiple deposit of alcohol solutions of uranium and thorium nitrates on aluminum electrodes and by the reduction of the nitrates to oxides through heating. The nonuniformity of the coating of the samples did not exceed 3%, as was determined by measuring the α -activity of different parts of the samples.

In order to calculate the dependence of the photofission on the energy of the γ -quanta it was necessary to measure the photofission yield and the intensity of the bremsstrahlung.

The photofission yield per unit of time $F(E)$ was measured as a function of the maximum energy of bremsstrahlung energy E . Measurements were taken at a distance of 1 m from the synchrotron target. Since photofission takes place under the action of photons with energies from threshold to maximum E the number

of photofissions $F(E)$ can be written as

$$F(E) = S n \int_0^E \sigma(W) N(W, E) dW, \quad (1)$$

where $\sigma(W)$ is the differential photofission cross section as a function of the photon energy W ; Sn is the number of fissioning nuclei in an area S of the sample; $N(W, E) dW$ is the number of photons with energies from W to $W + dW$ impinging per second on 1 cm^2 of the sample at maximum energy E .

Measurement of the absolute intensity of the γ -radiation was taken with a standard ionization chamber with 5 cm graphite walls. This chamber was used to determine the dependence of the ionization current $I(E)$ on the maximum γ -energy.

If $I(E)$ is the output current of the amplifier of the standard chamber and $\eta(E)$ is the efficiency of the chamber, which depends on the maximum energy of the γ -spectrum, then

$$I(E) = A \eta(E) \int_0^E W N(W, E) dW, \quad (2)$$

where A is a constant determined by the parameters of the standard chamber and of the amplifier.

To determine $\eta(E)$ we used the results in [5].

Control of the steadiness of γ -intensity during the course of operation was performed by a second ionization chamber with bakelite walls, placed in the path of the scattered radiation.

The spectrum of bremsstrahlung impinging on the standard chamber and on the sample may be represented as the product

$$N(W, E) dW = K(E) f(W, E) dW, \quad (3)$$

where $f(W, E) dW$ is the energy distribution of photons in the theoretical bremsstrahlung spectrum according to Heitler [6], and $K(E)$ is the fraction of the total flux impinging on the standard chamber which, in view of the dependence of the angular distribution of the bremsstrahlung on the maximum energy of the spectrum, is a function of E .

Using (1), (2) and (3), we obtain an expression determining the dependence of the differential fission cross section on the photon energy:

$$\frac{A}{Sn} \frac{F(E)}{I(E)} \eta(E) \int_0^E W f(W, E) dW = \int_0^E \sigma(W) f(W, E) dW. \quad (4)$$

This last relation is correct if the function $K(E)$ in the expression for $N(W, E)$ and the spectrum of the bremsstrahlung are identical in the expressions for the number of fissions (1) and for the current (2). This occurs if the bremsstrahlung spectrum is independent of the viewing angle with respect to the axis of the bremsstrahlung beam and if there is identical density of γ -radiation in the solid angles within which the standard chamber and the sample of fissioning material are viewed from the source of radiation. In our experiments both of these conditions were satisfied to a sufficient degree of accuracy.

Figure 1 shows the number of fissions per unit current in the standard chamber as a function of the maximum photon energy for uranium and thorium.

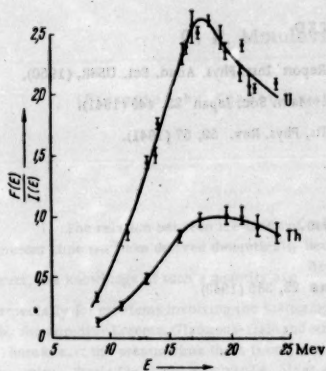


Figure 1.

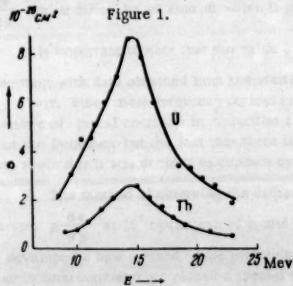


Figure 2.

By separate experiments it was shown that the number of fissions induced by neutrons is negligibly small compared with the number of photofissions.

It was also calculated that the probability of two fissions for one pulse of γ -emission of the synchrotron is less than 1% (with the resolving power of our apparatus two fissions would have been recorded as one).

The differential cross sections calculated from the data of Figure 1 are shown in Figure 2. The observed value of $\sigma(W)$ was corrected for adsorption of fragments in the uranium or thorium film. In the calculations we used a photon spectrum of the form $f(W, E)dW = dW/W$ (uniform spectrum). Under this hypothesis and with the parameters of our apparatus the cross section according to (4) is

$$\sigma(E) = \frac{AE}{\pi S} \frac{d}{dE} \left[\frac{F(E)}{I(E)} \eta(E) E \right] \quad (5)$$

Our curves of the energy dependence of the cross section show the resonance nature of photofission. The resonance energies for both uranium and thorium lie in the region of 14.5-15 Mev. The ratio of the photofission cross sections of thorium and uranium in the measured range of photon energies is of the order of 0.3. The maximum values of the uranium and thorium cross sections are $8.7 \cdot 10^{-28}$ and $2.6 \cdot 10^{-28}$ cm² respectively. The error in the determination of these cross sections is as much as 30% and is due essentially to inaccuracy in fitting the curves of photofission yield (Figure 1). The magnitude of the cross section obtained by subsequent differentiation of these curves is very sensitive to the method used in drawing them.

It is known [7] that the correct spectrum of bremsstrahlung does not agree completely with the uniform spectrum which we used for our calculations. We performed calculations employing Heitler's spectrum. The results of these calculations indicate that the qualitative picture of the dependence of the photofission cross section on photon energy is retained, but that the position of the maximum of the cross section is shifted in the lower energy range by approximately 2.5 Mev and that the magnitude of the cross section at the maximum increases by 15-20% compared with the results given above for a uniform spectrum.

The photofission cross section and its energy dependence which we obtained for uranium are in satisfactory agreement with the data of [3, 8].

At the time when we obtained our results there were no data in the literature regarding the dependence of thorium cross section on the γ -radiation energy.

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THE DENSITY DEPENDENCE OF THE INDEX OF REFRACTION AT LOW FREQUENCIES OF SOUND

G. P. Motulevich and I. L. Fabelinski

(Presented by Academician G. S. Landsberg July 19, 1955)

1. The relation between the index of refraction $n = \sqrt{\epsilon}$ and the density of a liquid has at the present time not been derived theoretically because of the unsatisfactory state of the theory of liquids. However, the knowledge of such a quantity as $\rho \frac{\partial \epsilon}{\partial \rho} = 2n \rho \frac{\partial n}{\partial \rho}$ is important for various physical problems, especially for problems involving the scattering of light, electrostriction, etc. Calculations of this quantity by the Lorentz - Lorentz, Gladstone-Dale and other formulas lead to results in disagreement with experiment. Therefore, at the present time there is only one correct method - the experimental determination of this quantity. Static [1-3] and dynamic [4, 5] (at a frequency of 10^7 cycles) experiments give values of $\rho \frac{\partial \epsilon}{\partial \rho}$ which differ by an amount which is greater than the limits of error of both experiments.

It is important to note that the value of $\rho \frac{\partial \epsilon}{\partial \rho}$ obtained at a frequency of 10^7 cycles is in good agreement with data obtained from the scattering of light in which frequencies of the order of 10^{10} cycles play a part. Since these frequency regions ($10^7 - 10^{10}$ c) are still far from the frequencies which are characteristic of optical constants in molecules and atoms ($10^{18} - 10^{19}$ c) one may assume that what is important is not the frequency but the fact that there is a transition from a static to a dynamic regime. To learn more about this it was decided to conduct dynamic measurements at a very low frequency (50 c).

2. The method of observing the diffraction of light by ultrasonic waves, which was used in [4, 5] to measure $\rho \frac{\partial \epsilon}{\partial \rho}$ at 10^7 cycles, can of course not be used to measure $\rho \frac{\partial \epsilon}{\partial \rho}$ at a low frequency. Therefore, we developed a new method whose principle is clear from the diagram in Figure 1. Between the plates of a Jamin interferometer we placed a special chamber made of a solid block of stainless steel perforated by two parallel channels closed by plane-parallel glass end plates. In one channel the density of the liquid was altered sinusoidally at 50 cycles by the diaphragm of an electromagnetic emitter. The length of the channel (20 cm) was much less than the wave length of sound in liquid for this frequency ($\Lambda = 30$ m). Therefore the optical path difference of rays passing through the channels also changed sinusoidally with the same frequency of 50 c. As the eye perceives an interference pattern averaged over time, a system of motionless bands is observed. The intensity of a point in the interference pattern is then given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \varphi J_0 \left[\frac{2\pi l \Delta \rho}{\lambda \rho} \left(\rho \frac{\partial n}{\partial \rho} \right) s \right], \quad (1)$$

where I_1 and I_2 are the intensities of light passing through the first and the second channel, φ is the phase difference in the absence of perturbations; λ is the wave length of the light; l is the length of a channel; $\Delta \rho / \rho$ is the relative change of density; and J_0 is the zero-th order Bessel function.

The interference term of (1) contains 2 factors $\cos \varphi$ and J_0 , of which $\cos \varphi$ determines the position of the dark and bright bands and J_0 determines the contrast range. With increase of the amplitude, the contrast at first diminishes and at a certain value the amplitude vanishes. The interference pattern is then replaced by uniform illumination. Upon further increase of the amplitude the contrast grows (the bands reappear), passes through a maximum and decreases again, etc. Contrast zeros, which are easily

observed experimentally, are given by values of the argument:

$$\frac{2\pi l \Delta p}{\lambda} \left(\rho \frac{\partial n}{\partial p} \right)_s = \alpha_l, \quad (2)$$

where α_l is the l -th root of the zero-th order Bessel function.

From Equation (2) it is possible to determine $\rho \frac{\partial \epsilon}{\partial p} = 2np \frac{\partial n}{\partial p}$, if $\frac{\Delta p}{\rho}$ is known at the same time for a given α_l . The determination of $\frac{\Delta p}{\rho}$ must be performed with great accuracy ($\approx 2\%$). For this purpose we developed a special method which will be described elsewhere.

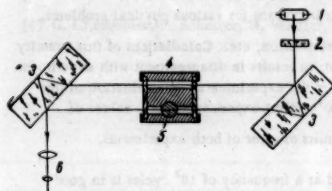


Figure 1. Diagram of the interference apparatus used to measure $\left(\rho \frac{\partial \epsilon}{\partial p} \right)_s$ at 50 cycles.

- 1) PRK-4 mercury lamp; 2) light filter; 3) plates of Jamin interferometer; 4) interference chamber; 5) electromagnetic emitter; 6) telescope.

3. The measurements of the adiabatic values of $\left(\rho \frac{\partial \epsilon}{\partial p} \right)_s$ at 50 cycles are given in Table 1, which also gives the wave lengths of the spectral lines which were used.

Table 2 gives a summary of the experimental values of $\rho \frac{\partial \epsilon}{\partial p}$ obtained at different sound frequencies for various substances.

The second column shows the isothermal values of $\left(\rho \frac{\partial \epsilon}{\partial p} \right)_T$ measured by a static method in [1, 2]. Column 3 gives the adiabatic values of $\left(\rho \frac{\partial \epsilon}{\partial p} \right)_s$ measured in the present work. Column 4 shows the isothermal values of $\left(\rho \frac{\partial \epsilon}{\partial p} \right)_T$ obtained

by calculation from the data of column 3 by using the equation in [4]:

$$\left(\rho \frac{\partial \epsilon}{\partial p} \right)_T - \left(\rho \frac{\partial \epsilon}{\partial p} \right)_s = -\frac{K}{1+K} \left[\left(\frac{1}{\alpha} \frac{\partial \epsilon}{\partial T} \right)_p + \left(\rho \frac{\partial \epsilon}{\partial p} \right)_s \right], \quad (3)$$

where $K = \frac{T\alpha^2}{c_p \beta s \rho}$; α is the coefficient of thermal expansion; β is the adiabatic compressibility; c_p is the specific heat at constant pressure.

Table 1

Measurements of the adiabatic value of $\left(\rho \frac{\partial \epsilon}{\partial p} \right)_s = 2n \left(\rho \frac{\partial n}{\partial p} \right)_s$ at 50 cycles.

Substance	λ, A	Series					
		1	2	3	4	5	6
Water	5780	—	—	0.90	0.885	0.91	0.90 ± 0.02
	5461	0.91	0.90	0.905	0.88	0.92	0.90 ± 0.015
	4358	—	—	0.94	0.89	0.94	0.92 ± 0.02
Benzine	5461	1.56	1.63	1.63	—	—	1.61 ± 0.06

Table 2

Values of $\rho \frac{\partial \epsilon}{\partial \rho} = 2np \frac{\partial n}{\partial \rho}$ obtained at various frequencies of sound *

Substance	Static value $\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_T$	Measurements at 50 c		Measurements at 10 ⁷ c		$\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_T$ from scattering of light (10 ¹⁰ c)
		$\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_S$	$\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_T$	$\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_S$	$\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_T$	
1	2	3	4	5	6	7
Water	0.868	0.90	0.90	0.79	0.80	0.80
Methyl alcohol	0.902	—	—	0.79	0.80	0.81
Ethyl alcohol	1.041	—	—	0.87	0.90	0.97
Ether	0.989	—	—	0.88	0.90	0.91
Benzene	1.62	1.61	1.61	1.46	1.51	1.51
Toluene	1.60	—	—	1.515	1.54	1.525

*The static values (column 2) refer to the wave length of light $\lambda = 5893 \text{ \AA}$, with the exception of toluene for which the values given refer to $\lambda = 5461 \text{ \AA}$. The measurements at 50 c (columns 3 and 4) refer to light of $\lambda = 5461 \text{ \AA}$, since at this wave length the accuracy of the measurements is maximal. Finally, the values at 10⁷ c and also the measurements of $\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_T$ from light scattering refer to $\lambda = 4358 \text{ \AA}$. Studies of the dependence of $\rho \frac{\partial \epsilon}{\partial \rho}$ on the wave length of the light which are given in [1, 4] as well as in the present work show that this dependence is very insignificant.

Column 5 gives the adiabatic values of $\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_S$ measured by the authors at 10⁷ c [4]. Column 6 gives the isothermal values of $\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_T$ as calculated from the same formula. Column 7 gives the isothermal values of $\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_T$ obtained in experiments on light scattering [4]; these last measurements refer to a sound frequency $\approx 10^{10} \text{ c}$.

From Table 2 it follows that for all the substances investigated dispersion of the quantity $\rho \frac{\partial \epsilon}{\partial \rho}$ is observed in the range 50 - 10⁷ c. The value of $\rho \frac{\partial \epsilon}{\partial \rho}$ at 50 c agrees with the static values and the values at 10⁷ c agree with the values for hypersound (10¹⁰ c).

In theory the values of $\rho \frac{\partial \epsilon}{\partial \rho}$ measured by the static and dynamic methods may differ if it is considered that this quantity is a function with a certain range. This range must have an order of magnitude determined by intermolecular interactions. Taking this into account, we may state that for measurement of $\rho \frac{\partial \epsilon}{\partial \rho}$ at a sonic frequency corresponding to sonic wave length Λ ,

$$\rho \frac{\partial \epsilon}{\partial \rho} = \left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_0 \left\{ 1 + k \left(\frac{a}{\Lambda}\right)^3 \right\}, \quad (6)$$

where $\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_0$ is the static value of this quantity; a is a constant with the dimension of a length and equal in order of magnitude to $\approx 10^{-7} \text{ cm}$; k is a dimensionless constant ≈ 1 , whose sign is not determined beforehand. For frequencies of 50, 10⁷ and 10¹⁰ cycles the additional term in the brackets is of the order of 10⁻¹⁰, 10⁻¹⁰ and 10⁻⁴ respectively. Thus consideration of these circumstances does not explain the

effect and the problem needs further theoretical study.

In conclusion we wish to thank G. S. Landsberg for valuable advice and discussions.

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ON THE SPECTRA OF PROTONS AND π^- -MESONS AT SEA LEVEL

L. I. Potapov and N. V. Shostakovich*

(Presented by Academician A. I. Alikhanov July 21, 1955)

The present work was undertaken for the purpose of studying the momentum spectra of protons and π^- -mesons at sea level. Measurements were carried out with a magnetic spectrometer of the Alikhanov-Alikhanov type. A magnetic field of 7500 oersteds was excited by an electromagnet whose poles were 58 cm long and 20 cm wide. The pole gap was 10 cm.

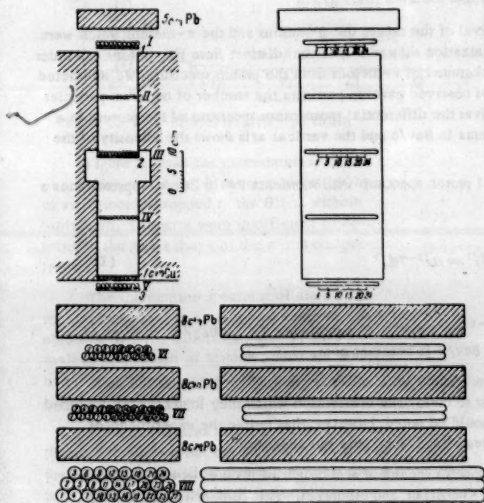


Figure 1. Diagram of the apparatus.

absorbers each 8 cm thick. Above 3 and V combined in one tray was a 1 cm copper absorber and above the entire setup at a distance of 5 cm from the upper edge of the poles was a 5 cm block of lead.

The momentum of a charged particle was determined from its curvature in the magnetic field. Inaccuracy of this measurement was due, first, to the finite geometrical dimensions of the counters, and, second, to Coulomb scattering of the particles from the counter walls. The calculated root mean square error resulting from the combination of these two errors was for $P = 0.5 \text{ Bev}/c - 9.7\%$; for $P = 1 \text{ Bev}/c - 16\%$; and for $P = 5 \text{ Bev}/c - 80\%$.

* Deceased.

Three variations of the experiment were performed, differing only in the number and arrangement of the counters in layers below the poles of the magnet. The areas of the layers remained unchanged. Figure 1 shows the last of the variants.

8 banks of counters were placed in the gap of the magnet. Rows 1, 2 and 3, which we call "coordinate", consisted of counters with copper cathodes of 4.8 mm diameter, which determined the trajectory of a particle in a plane perpendicular to the direction of the magnetic field. Banks I-V consisted of counters with aluminum and copper cathodes of 8 and 10 mm diameters, which determined the trajectory in a plane parallel to the direction of the magnetic field. The walls of the counters were 0.1 mm thick. Below the poles, in order to be able to trace the trajectory of a particle after it left the magnetic field, three more banks of counters were placed, of which banks VI and VII each consisted of two rows of counters placed close to each other, with a diameter of 20 mm, and bank VIII consisted of three rows of counters with a 30 mm diameter.

Between VI, VII and VIII were three lead

In treating the experimental data we selected only those trajectories which described a circle in a plane perpendicular to the direction of the magnetic field and a straight line in a plane parallel to the direction of the magnetic field, and also: 1) are recorded once in each of the coordinate rows; 2) are

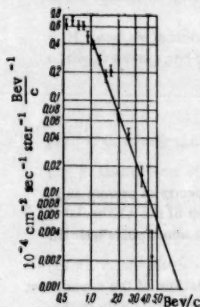


Figure 2. Proton momentum spectrum.

are recorded once in not less than three of I-V; 3) do not strike the outer counters of VI (counters 1, 2, 16, 17) or VII (counters 1, 2, 22, 23). In our apparatus instances were possible in which a particle by interacting with the material of the absorber, above VI for example, was recorded in VI but was not recorded in VII for two reasons: either it was stopped in the absorber above VII or it left the apparatus at an angle which took it outside VII. In the first instance the particle satisfied our criterion for selection of trajectories, in the second instance it did not satisfy the criterion. Such trajectories were not included in the spectrum given below. In the momentum range of $1 \text{ BeV}/c < P < \infty$, 27% of such cases were observed of the total in Table 1.

Table 1 gives the results of the measurements. In 2220 hours we recorded 794 positive and 117 negative particles with momentum above $0.5 \text{ BeV}/c$, which were stopped in the filters placed between V and VI and between VI and VII and corresponding to ranges $8.9 \leq R \leq 189.7 \text{ g}/\text{cm}^2$.

In each interval of the ranges the μ -mesons and the π -mesons which were stopped through ionization showed a spectrum distinct from the protons. In order to exclude the background of π -mesons from the proton spectrum, we subtracted from the number of observed positive particles the number of negative particles

in each respective range of momenta. Figure 2 gives the differential momentum spectrum of the protons in a log-log plot. The horizontal axis shows the momenta in BeV/c and the vertical axis shows the intensity of the vertical proton flux per unit momentum.

As can be seen from Figure 2 the differential proton spectrum with momenta $P > 1.0 \text{ BeV}/c$ approximates a power function of the form

$$N(P)dP = aP^{-\gamma}dP \quad (1)$$

with $\gamma = 2.8 \pm 0.02$ and $a = (0.69 \pm 0.049) \cdot 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$. For momenta $P < 1.0 \text{ BeV}/c$ the slope of the curve gradually decreases and for $P = 0.6 - 0.7 \text{ BeV}/c$ it becomes zero.

The proton momentum distribution at sea level which we found is in good agreement with the data of Mylroil and Wilson [1] who used a method similar to ours. The distribution which they found is also described by a function of the form of (1) with $\gamma = 2.8$. It should be noted, however, that unlike the experiments of these authors, in our experiments a 5 cm block of lead was placed over the apparatus.

We also compared our proton spectrum with spectra measured at a height of 3200 m with a magnetic spectrometer by A. Dadaian and G. Merzon [2], N. Kocharian and others [3]. This comparison shows that the form of the spectrum and the exponent at sea level and at 3200 m within the limits of errors of measurement do not differ.

The integral intensity of the vertical proton flux with momenta $P > 1.0 \text{ BeV}/c$ under 5 cm of lead is

$$I_0 = (0.384 \pm 0.027) \cdot 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$$

If we take into consideration those protons which were not included in the spectrum of Figure 2 in accordance with our selection criterion (their number comprises 33% of that indicated in Table 1) we obtain

$$I_0 = (0.506 \pm 0.035) \cdot 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$$

Mytroll and Wilson obtained for the integral intensity of the vertical proton flux in air at sea level the value $0.65 \cdot 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$. Making the appropriate conversion for comparison of our results with the results of Mytroll and Wilson we obtain $I_0 = (0.465 \pm 0.033) \cdot 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$, and allowing for the cases not included in the spectrum of Figure 2, we have $I_0 = (0.61 \pm 0.04) \cdot 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$. Within the limits of errors of measurements our results may be considered to agree with the results obtained by Mytroll and Wilson.

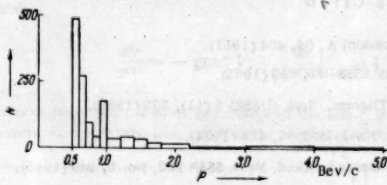


Figure 3. Spectrum of negative particles.

Figure 3 shows the momentum spectrum of negative particles which were identified by us as π^- -mesons stopped in the filters without ionization. Our data were insufficient to determine the exact shape of the π^- -meson spectrum.

The momentum spectrum of the π^- -mesons generated in lead by fast neutrons was obtained by A. I. Alkhaniyan and V. Sh. Kamalian at a height of 3200 m above sea level; they found that it approximates a power function with the exponent $\gamma = 3 \pm 0.2$.

From our data as given in Table 1 and in Figure 3 it follows that the number of π^- -mesons decreases sharply with increasing momentum. A trial determination of the differential momentum spectrum of π^- -mesons indicates that it closely approximates the function (1) with exponent $\gamma \sim 3$. In the momentum range close to 1 BeV/c an increase of the number of particles is observed which cannot be explained by statistical fluctuations.

G. Eliseev, V. Liubimov and V. Kosmachevskii in measuring the momentum and ionizing power of particles trapped in filters as a result of nuclear collisions observed an anomalous rise in the momentum spectrum of

Table 1

Momentum range, 10^4 ev/c	Number of particles observed				Number of protons	Number of protons with transmission correction	Number of protons with probability of absorption	Number of protons per interval Bev/c	I_0 , $10^{-4} \text{ cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$	Number of negative particles with transmission correction per interval BeV/c
	Positive		Negative							
	R_{π^+} g/cm ²	R_{π^+} g/cm ²	R_{π^-} g/cm ²	R_{π^-} g/cm ²						
0.5-0.6	90	29	119	8	28	36	112.4±14.5	1124±145	0.64±0.082	487±25.4
0.6-0.7	94	16	410	7	13	20	112.4±14.5	1220±133	0.694±0.076	270±19
0.7-0.8	75	13	388	1	6	7	110±11.3	1100±133	0.625±0.064	95±11.2
0.8-0.9	60	23	83	1	2	3	108±10.8	1080±108	0.614±0.062	176±15.4
0.9-1.0	37	23	83	1	6	13	85±11.1	850±110	0.391±0.035	34±6.8
1.0-1.2	42	24	76	2	3	6	66±10.4	137±12.5	0.271±0.030	40±5.7
1.2-1.4	27	28	76	2	3	6	66±10.4	137±12.5	0.82±0.026	34±6.8
1.4-1.6	19	19	38	3	5	5	64±9.1	477±52	0.0715±0.011	17±4.7
1.6-1.8	24	17	41	2	2	3	51±8.2	369±46	0.046±0.006	5±2.6
1.8-2.2	15	16	31	2	2	2	50.4±8.4	126±21	0.016±0.004	8±3.4
2.2-3.0	14	21	35	2	4	3	62.8±7.2	78±11	0.0022±0.0019	2.7±1.9
3.0-4.0	10	11	21	2	4	6	28.4±7.2	28.4±7.2		
4.0-5.0	1	1	2	2	2	2	3.9±3.4	3.9±3.4		
>5.0	8	9	17	1	1	2	27±6.2			

π -mesons in the range $7-12 \cdot 10^8$ ev/c. Having determined the mass of the particles in the anomalous group from the ionization and momentum these authors concluded that the particles are heavy mesons of mass about $900 m_e$. It is possible that the group of particles which we observed in the 1 Bev/c region also consists of heavy mesons.

In conclusion we wish to acknowledge our deep gratitude to A. I. Alkhanyan for his sustained interest and for valuable discussions.

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THE MEASUREMENT OF ION MASSES BY THE USE OF ELECTRIC FIELDS

V. B. Fiks

(Presented by Academician A. A. Lebedev July 21, 1955)

In an electric field it is impossible to separate ions with identical initial energies according to their masses. This well-known fact follows directly from the equations of motion when written in the appropriate form:

$$\frac{d^2r}{dt^2} = -\text{grad } V; \quad \left| \frac{dr}{dt} \right|_{t=0} = \sqrt{2V_0} = \text{const}; \quad \tau = \sqrt{\frac{e}{m}} t. \quad (1)$$

Here m and e are the mass and charge of the ion; t is the time; and $V_0 = \epsilon_k(0)/e$; $\epsilon_k(0)$ is the initial kinetic energy of the ion; and V is the potential of the electric field.

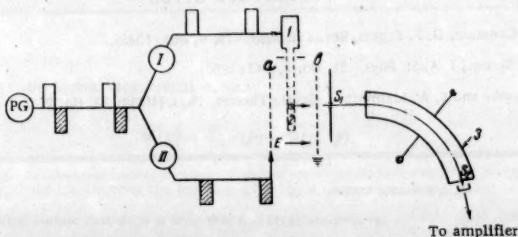


Figure 1. Diagram of the mass analyzer.

From the equations and the initial conditions it is easily seen that the trajectories of monoenergetic ions are independent of the mass. But the time required for the ions to move along these trajectories does depend on the mass, as follows from (1), and this dependence may be made the basis of measurements. Existing nonmagnetic mass-analyzers are based essentially on the direct or indirect measurement of the time of flight of ions along a given path [1-3]. There is another way of carrying out the measurements.

It is possible in an electric field to separate according to mass ions possessing identical initial momentum. In this case, the energy of the ions depends on the mass and, consequently, the direct or indirect measurement of the energy by the use of an electrostatic field leads to a measurement of mass.

Mass spectrometers operated in this way must consist of two parts: a source producing ions with identical momenta, and an analyzing electric field.

Let us examine the diagram of the ion source (see Figure 1). The parallel grids a and b form a plane condenser. During a short time interval t_1 the electron beam from the gun 1 forms an ion packet 2, between the grids. After switching off the electron gun, an accelerating voltage pulse of duration t_1 is applied to the grids a and b.

If the time required for the leading edge of the ion packet to reach grid b is longer than the duration

of the accelerating pulse all ions between the grids will acquire identical momentum P :

$$P = \int_0^{t_1} eE(t) dt = e\bar{E}t_1 = \text{const};$$

where $E(t)$ and $\bar{E}$ are, respectively, the instantaneous and average values of the electric field strength.

For mass analysis of ion packets electrostatic analyzers of various types can be used. We consider, for example, the complete scheme of the instrument with a cylindrical condenser as a mass analyzer. The pulse generator PG develops pulses whose shape is shown in Figure 1. The pulses are separated in two rectifiers I and II. The first pulse reaches the electron gun and turns on the electron beam. The second pulse applies an accelerating field between grids a and b .

It is also possible to use a sinusoidal wave generator. Ions with identical momenta enter the Hughes-Rojansky condenser [3] which focuses them according to their directions and analyzes them according to their energies and, consequently, according to their masses. Behind the exit slit S_2 , there is a receiver connected to a direct current amplifier.

Other types of electrostatic analyzers can also be used to analyze ion masses.

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EXPANSION OF A WAVE FUNCTION IN IRREDUCIBLE REPRESENTATIONS OF THE LORENTZ GROUP

I. S. Shapiro

(Presented by Academician L. D. Landau July 16, 1955)

We shall examine the transformation properties in a momentum representation of wave functions which describe the state of a single particle with zero spin. This function will be denoted by $\psi(p)$ where p is the four-dimensional momentum of the particle, whose mass will be assumed to be unity:

$$-p^2 = p_\mu p_\mu = 1, \quad \mu = 1, 2, 3, 4; \quad p_4 = ip_0; \quad (1)$$

p_0 is the energy of the particle (we use a system of units in which $\hbar = c = 1$).

$|\psi(p)|^2 d^3p/p_0$ is the number of particles in the invariant phase space d^3p/p_0 .

If

$$p' = g p, \quad (2)$$

where g is the matrix of the Lorentz transformation, then

$$\psi'(p) = \hat{T}_g \psi(p) = \psi(g^{-1}p). \quad (3)$$

Here $\hat{T}_g$ denotes an operator transforming the function $\psi(p)$ by a Lorentz rotation g .

We shall hereafter assume that $\psi(p)$ is such that the invariant integral

$$\int |\psi(p)|^2 p_0^{-1} d^3p, \quad (4)$$

which represents the total number of particles, exists. Representations of the Lorentz group (3) which transforms the functions $\psi(p)$ are unitary, as follows from the existence of (4), and may in general be reduced. In this connection there arises the problem of an expansion in irreducible representations of the Lorentz group. An expansion in irreducible representations of the Lorentz group establishes a relativistically invariant classification of the functions, which is analogous to the way in which an expansion in irreducible representations of the group of three-dimensional rotations leads to a classification of states according to angular momentum that is invariant with respect to this group.

A representation of (3), which due to the existence of the integral (4) is unitary, can be expanded only into unitary (and therefore infinite-dimensional) irreducible representations of the Lorentz group. A solution of our problem may be obtained by applying to a given concrete case the results of the general theory of unitary representations of the Lorentz group which was developed by I. M. Gelfand and M. A. Naimark [1-3]. Each irreducible unitary representation is characterized by two numbers (m, ρ) where m is an integer and ρ may be any real number (representations of the so-called basic series) or a purely imaginary number subject to the condition $|\rho| < 2$. Our representation appears to resolve only into irreducible representations of the basic series so that the number ρ hereafter is real. The numbers m, ρ define the eigenvalues of the four-dimensional momentum

operator

$$\hat{F} = \frac{1}{2} \hat{M}_{\mu\nu} \hat{M}_{\nu\mu} \quad (5)$$

and of the operator

$$\hat{G} = \varepsilon_{\mu\nu\lambda} \hat{M}_{\mu\nu} \hat{M}_{\lambda\lambda}. \quad (6)$$

Here $\varepsilon_{\mu\nu\lambda}$ is the completely antisymmetrical unit tensor of the fourth rank, and $M_{\mu\nu}$ are operators of the components of the four-dimensional angular momentum ($\mu, \nu = 1, 2, 3, 4$). The operators $\hat{F}$ and $\hat{G}$ commute with all the operators $\hat{M}_{\mu\nu}$ that agree to within the factor 1 with the operators of infinitely small rotations. Hence it follows that for irreducible representations the operators $\hat{F}$ and $\hat{G}$ must be simply operators which multiply by a number. Thus

$$\hat{F} = \left(1 - \frac{m^2 - \rho^2}{4}\right), \quad \hat{G} = m\rho. \quad (7)$$

In our case of scalar functions the operators $\hat{M}_{\mu\nu}$ have the simple form

$$\hat{M}_{\mu\nu} = -i \left(p_\mu \frac{\partial}{\partial p_\nu} - p_\nu \frac{\partial}{\partial p_\mu} \right). \quad (8)$$

Using (6), it is easily shown that in this case

$$\hat{G} \equiv 0. \quad (9)$$

From (6) and (9) it follows that for irreducible representations composing the representation of (3) at least one of the numbers m, ρ must be zero. A calculation shows that m is zero. The reason for this is that the function $\psi(p)$ depends on a time-like vector argument. Functions of a spacelike four-dimensional vector ($p^2 = +1$) can be transformed both by a representation with $m = 0$ and by a representation with $\rho = 0$. (This last was shown by V. L. Ginsburg and M. E. Tamm [4]: their result refers to functions which on being expanded in series of Laplacian spherical harmonics do not contain terms which are independent of the angles).

The use of equations (151) and (152) of [1] and of §§ 7, 8, 19 and 24 of [3] makes it possible to obtain the following final results

$$\psi(p) = \left(\frac{1}{4\pi}\right)^{1/2} \int C(n, p) (n_\mu p_\mu)^{-1+i\rho/2} d\Omega_n p^2 dp; \quad (10)$$

$$C(n, p) = \left(\frac{1}{4\pi}\right)^{1/2} \int \psi(p) (n_\mu p_\mu)^{-1-i\rho/2} p_\mu^{-1} d^2 p; \quad (11)$$

$$\int |C(n, p)|^2 d\Omega_n p^2 dp = \int |\psi(p)|^2 p_0^{-1} d^2 p. \quad (12)$$

Here n_μ is the isotropic unit of the vector

$$n^2 = 1, \quad n_4 = -i, \quad n_\mu n_\mu = 0; \quad (13)$$

and $d\Omega_n$ is the element of solid angle in the space of vectors n ; integration with respect to p goes from 0 to ∞ . The function $C(n, p)$ is transformed by an irreducible unitary of the type $(0, \rho)$. The number ρ determines the type of the representation and therefore remains unchanged for all Lorentz transformations.

The unit isotropic vector n_μ is also transformed like k_μ/k_0 where k_μ is the momentum of a photon and k_0 is its frequency. In particular if we transform to a system moving with velocity β , along the Z-axis, the function $C(\mathbf{n}, \rho)$ is transformed by the rule

$$C(\mathbf{n}, \rho) \rightarrow \left\{ \frac{1 - \beta \cos \vartheta}{1 - \beta^2} \right\}^{-1/2} C(\mathbf{n}', \rho), \quad (14)$$

where the polar angles ϑ', φ' of the vector $\mathbf{n}'$ are related to the polar angles ϑ, φ of the vector $\mathbf{n}$ by:

$$\cos \vartheta' = \frac{1 - \beta + \cos \vartheta}{1 - \beta \cos \vartheta}, \quad \varphi' = \varphi \quad (15)$$

this result is easily obtained from (11) by transforming $\psi(p)$ in accordance with (3) and setting

$$(g^{-1}p)_1 = p_1, (g^{-1}p)_2 = p_2, (g^{-1}p)_3 = \frac{p_3 - i\beta p_4}{\sqrt{1 - \beta^2}}, (g^{-1}p)_4 = \frac{p_4 + i\beta p_3}{\sqrt{1 - \beta^2}}.$$

According to (10)-(13) a given $C(\mathbf{n}, \rho)$ completely determines the initial function $\psi(p)$. Therefore by the expansion (10) a state of the particle described by $\psi(p)$ is represented as a superposition of states with given quantum number ρ . A classification of the states according to ρ is obviously relativistically invariant since ρ characterizes the transformation properties of $C(\mathbf{n}, \rho)$ with respect to the Lorentz group.

We have considered wave functions of particles with spin zero. In the case of non-zero spin $\frac{1}{2}$, the representation by which the wave function is transformed reduces to irreducible representations of the type $(2s, \rho)$.

In conclusion I wish to express my deep gratitude to Academician L. D. Landau, and also to V. L. Ginsburg, I. M. Gel'fand and M. A. Naimark for their valuable comments during discussions.

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ON THE RELATIONS BETWEEN CROSS SECTIONS WHICH RESULT FROM THE HYPOTHESIS OF ISOTOPIC INVARIANCE

N. Dushin and I. Shmushkevich

(Presented by Academician L. D. Landau August 31, 1955)

In [1] a simple method was given for obtaining the relations between cross sections which result from the hypothesis of isotopic invariance. In some cases, however, there are more relations than were given in [1]. We shall show below that a small change of the procedure employed in [1] easily gives these additional relations.

1. Let us consider, for example, the collisions of π -mesons with nucleons which are accompanied by a transformation of the π -particle into two π -particles. We shall assume an equal number of protons and neutrons in the target and an equal number of π^+ -, π^- -, and π^0 -mesons in the beam and we write out all possible processes of the type in which we are interested:

- | | |
|--|---|
| 1) $\pi^+ + p \rightarrow p + \pi^+ + \pi^0$; | 1') $\pi^- + n \rightarrow n + \pi^- + \pi^0$; |
| 2) $\pi^+ + n \rightarrow n + \pi^+ + \pi^0$; | 2') $\pi^- + p \rightarrow p + \pi^- + \pi^0$; |
| 3) $\pi^+ + n \rightarrow p + \pi^0 + \pi^0$; | 3') $\pi^- + p \rightarrow n + \pi^0 + \pi^0$; |
| 4) $\pi^+ + n \rightarrow p + \pi^+ + \pi^-$; | 4') $\pi^- + p \rightarrow n + \pi^- + \pi^-$; |
| 5) $\pi^+ + p \rightarrow n + \pi^+ + \pi^-$; | 5') $\pi^- + n \rightarrow p + \pi^- + \pi^-$; |
| 6) $\pi^0 + p \rightarrow p + \pi^0 + \pi^0$; | 6') $\pi^0 + n \rightarrow n + \pi^0 + \pi^0$; |
| 7) $\pi^0 + p \rightarrow p + \pi^+ + \pi^-$; | 7') $\pi^0 + n \rightarrow n + \pi^+ + \pi^-$; |
| 8) $\pi^0 + n \rightarrow p + \pi^- + \pi^0$; | 8') $\pi^0 + p \rightarrow n + \pi^+ + \pi^0$; |

From the charge symmetry of the theory it follows that the cross sections of the reactions which are denoted by the same number (with and without a prime) are identical.

Compiling a table by the method described in [1] we obtain (see Table 1, a)

$$\sigma_1 + \sigma_2 + \sigma_3 + 4(\sigma_4 + \sigma_5) = 2(\sigma_6 + \sigma_7 + \sigma_8). \quad (1)$$

Table 1

	a			b		
	π^+	π^-	π^0	π^+	π^-	π^0
σ_1	1	1	2	1	1	0
σ_2	1	1	2	1	1	0
σ_3	0	0	4	1	1	0
σ_4	2	2	0	1	1	0
σ_5	2	2	0	1	1	0
σ_6	0	0	4	0	0	2
σ_7	2	2	0	0	0	2
σ_8	1	1	2	0	0	2

We now reverse the discussion in [1] concerning processes that are the inverse of 1), 1'), ..., 8'), i.e., trinary collisions of two π -mesons and one nucleon leading to the formation of one nucleon and one π -meson. We assume that each of the three beams of colliding particles is isotopically unpolarized, i.e., that there is an equal number of protons and neutrons in the nucleon beam, and that there is an equal number of π^+ -, π^- -, and π^0 -mesons in each of the meson beams. Under these conditions the beams of created π -mesons and nucleons will also be isotopically unpolarized. Therefore by equating the total number of π^+ -, π^- -, and π^0 -mesons created in both

processes we obtain a relation between the probabilities of the various inverse processes. Thus as a supplement to (1) we will obtain one more relation between the cross sections of the direct processes since the probabilities of the inverse processes are proportional to the cross sections of the corresponding direct processes (we are referring to the total cross sections averaged over the usual spins). This relation is most easily obtained by

using Table 1, b. This table gives the total number of π^+ -, π^- -, and π^0 -mesons in the initial states of all direct processes which have the same cross section (these totals obviously agree with the corresponding totals of π -mesons in the final states of the inverse processes). We now multiply the numbers in the column for π^+ -mesons (or for π^- -mesons) of Table 1, b by the corresponding cross sections and add the resulting products. Denoting the result by n^+ we have $n^+ = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \sigma_5$. The analogous result for π^0 -mesons is $n^0 = 2\sigma_6 + 2\sigma_7 + 2\sigma_8$.

From the condition $n^+ = n^- = n^0$, we find

$$\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \sigma_5 = 2(\sigma_6 + \sigma_7 + \sigma_8). \quad (2)$$

We shall examine a few more processes and shall find relations between the cross sections by the above-described method.

2. The scattering of π -mesons by deuterons. In the scattering of π -mesons by deuterons accompanied by the splitting of the deuteron the following processes are possible:

- | | |
|--|---|
| 1) $\pi^+ + d \rightarrow p + n + \pi^+$; | 1') $\pi^- + d \rightarrow n + p + \pi^-$; |
| 2) $\pi^+ + d \rightarrow p + p + \pi^0$; | 2') $\pi^- + d \rightarrow n + n + \pi^0$; |
| 3) $\pi^0 + d \rightarrow p + p + \pi^-$; | 3') $\pi^0 + d \rightarrow n + n + \pi^+$; |
| 4) $\pi^0 + d \rightarrow p + n + \pi^0$; | |

Table 2

	a			b		
	π^+	π^-	π^0	π^+	π^-	π^0
σ_1	1	1	0	1	1	0
σ_2	0	0	2	1	1	0
σ_3	1	1	0	0	0	2
σ_4	0	0	1	0	0	1

Using Tables 2, a and 2, b we find the relations

$$\sigma_1 + \sigma_3 = 2\sigma_2 + \sigma_4, \quad (3)$$

$$\sigma_1 + \sigma_2 = 2\sigma_3 + \sigma_4. \quad (4)$$

From these there follows

$$\sigma_2 = \sigma_3, \quad (5)$$

$$\sigma_1 = \sigma_3 + \sigma_4. \quad (6)$$

In elastic scattering of π -mesons by deuterons, processes (2) and (3) are impossible. Therefore

$$\sigma_1(\pi^+ + d \rightarrow \pi^+ + d) = \sigma_4(\pi^0 + d \rightarrow \pi^0 + d). \quad (7)$$

3. The creation of two π -mesons in collisions of π -mesons with deuterons. In this case the following processes are possible:

- | | |
|--|---|
| 1) $\pi^+ + d \rightarrow p + n + \pi^+ + \pi^0$; | 1') $\pi^- + d \rightarrow n + p + \pi^- + \pi^0$; |
| 2) $\pi^+ + d \rightarrow p + p + \pi^0 + \pi^0$; | 2') $\pi^- + d \rightarrow n + n + \pi^0 + \pi^0$; |
| 3) $\pi^+ + d \rightarrow p + p + \pi^+ + \pi^-$; | 3') $\pi^- + d \rightarrow n + n + \pi^- + \pi^+$; |
| 4) $\pi^+ + d \rightarrow n + n + \pi^+ + \pi^+$; | 4') $\pi^- + d \rightarrow p + p + \pi^- + \pi^-$; |
| 5) $\pi^0 + d \rightarrow p + n + \pi^0 + \pi^0$; | |
| 6) $\pi^0 + d \rightarrow p + n + \pi^+ + \pi^-$; | |
| 7) $\pi^0 + d \rightarrow p + p + \pi^- + \pi^0$; | 7') $\pi^0 + d \rightarrow n + n + \pi^+ + \pi^0$. |

With the aid of Tables 3, a and 3, b, we obtain

$$2\sigma_3 + 2\sigma_4 + \sigma_6 = \sigma_1 + 4\sigma_2 + 2\sigma_5 + 2\sigma_7, \quad (8)$$

$$\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 = \sigma_6 + \sigma_8 + 2\sigma_9. \quad (9)$$

For processes in which the deuteron does not split, reactions (2), (3), (4) and (7) are impossible. Therefore, for these processes from (8) and (9) there follows

$$\sigma_s = \sigma_1 + 2\sigma_2, \quad \sigma_1 = \sigma_3 + \sigma_4.$$

Consequently,

$$\sigma_s(\pi^0 + d \rightarrow d + \pi^0 + \pi^0) = 0; \quad (10)$$

$$\sigma_1(\pi^+ + d \rightarrow d + \pi^+ + \pi^0) = \sigma_s(\pi^0 + d \rightarrow d + \pi^+ + \pi^-). \quad (11)$$

4. The decay of τ -mesons into three π -mesons. Under the assumption that the isotopic spin of τ -mesons is unity we can consider the following decays:

- 1) $\tau^+ \rightarrow \pi^+ + \pi^0 + \pi^0$; 1') $\tau^- \rightarrow \pi^- + \pi^0 + \pi^0$;
- 2) $\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^-$; 2') $\tau^- \rightarrow \pi^- + \pi^- + \pi^+$;
- 3) $\tau^0 \rightarrow \pi^0 + \pi^0 + \pi^0$;
- 4) $\tau^0 \rightarrow \pi^+ + \pi^- + \pi^0$;

The first relation between the decay probabilities is obtained by using Table 4, a:

$$w_2 = w_1 + w_3. \quad (12)$$

Table 3

	a			b		
	π^+	π^-	π^0	π^+	π^-	π^0
σ_1	1	1	2	1	1	0
σ_2	0	0	4	1	1	0
σ_3	2	2	0	1	1	0
σ_4	2	2	0	1	1	0
σ_5	0	0	2	0	0	1
σ_6	1	1	0	0	0	1
σ_7	1	1	2	0	0	2

This relation agrees with those obtained previously (see [1, (8)]). Table 4, b gives the total numbers of τ^+ , τ^- , and τ^0 -mesons in the initial states of those processes in which decay takes place with equal probability. Since in the inverse processes where all three beams of π -mesons are isotopically unpolarized, the τ^+ , τ^- , and τ^0 -mesons must be created in equal numbers, by using Table 4, b, we obtain as a supplement to (12) a second relation

$$w_1 + w_2 = w_3 + w_4. \quad (13)$$

Table 4

	a			b		
	π^+	π^-	π^0	π^+	π^-	π^0
w_1	1	1	4	1	1	0
w_2	3	3	0	1	1	0
w_3	0	0	3	0	0	1
w_4	1	1	1	0	0	1

From (12) and (15) we find that

$$w_4 = 2w_1. \quad (14)$$

We note in conclusion that for such processes as the scattering of π -mesons by nucleons ([1, (1)] or elastic scattering of π -mesons by deuterons (see Equation (7) of this article) there is altogether only one relation. This is associated with the obvious fact

that in these processes there are equal sets of particles in the initial and final states (although possibly in different charge states). Only one relation is also obtained for processes in which in the initial (final) state there are only particles with isotopic spins equal either to zero or one-half (for example, only nucleons and deuterons), and in the final (initial) state, in addition to such particles, there are also particles with isotopic spin one (for example, π -mesons). All processes, except the last (decay of a τ -meson) of those considered in [1] refer to processes of one of these two types, and therefore for these processes there is only one relation between the cross sections, which is given in [1]. If in addition to particles with zero or one-half isotopic spin particles of different types participate in the reaction with isotopic spin equal to or larger than unity then

number of relations between cross sections will be equal to or larger than two.*

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Figure 1. Ratio of cross sections of samples of different sizes. 1) $R = 0.5$; 2) $R = 0.3$; 3) $R = 0.1$. The curves are calculated for $T = 20^\circ\text{C}$ and $T = 40^\circ\text{C}$.

*In this connection see the last remark in [1].

HEAT CAPACITY OF LAYER LATTICES AT LOW TEMPERATURES

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(Presented by Academician P. L. Kapitza July 27, 1955)

We have investigated the temperature dependence of the heat capacity of CdI_2 , CdBr_2 and CdCl_2 , which have a layer lattice structure. The forces between the atoms in the layers and between the layers are of two different types: within the layers ionic binding exists, and between the layers there are van der Waals forces. It was this fact that interested us in the cadmium halides. Figure 1 shows the temperature dependence of the heat capacities of CdI_2 , CdBr_2 and CdCl_2 in the most interesting region, which is below 60°K. A portion of the region of linear dependence can be seen which broadens considerably as we go from the iodide to the chloride; in CdCl_2 this portion is very much extended. At lower temperatures the temperature dependence becomes stronger and is transformed into a square law dependence; in the case of CdI_2 , this transition takes place at 17-18°K; for CdBr_2 at 15-16°K; and for CdCl_2 at 13-14°K. The straight line extrapolations of the curves showing the dependence of C_p on T^2 intersect the negative ordinate axis at $T = 0$. This shows that as the temperature decreases the temperature dependence is of a power higher than the second. We may assume that at sufficiently low temperatures the heat capacity is proportional to T^3 .

cal./mole·deg.

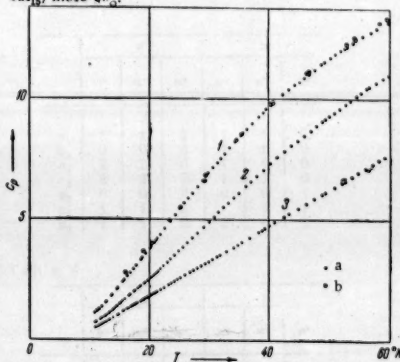


Figure 1. Heat capacity of cadmium halides between 11° and 60°K.

1) CdI_2 ; 2) CdBr_2 ; 3) CdCl_2 .

a) Our measurements; b) measurements of Dworkin et al. [2].

Our results are not in disagreement with the theoretical picture [1].

Dworkin, Sasmor and Artsdalen [2] measured the heat capacity of CdI_2 between 302 and 16°K and found that at the lower limit C_p changes proportionately to $T^{1.4}$. A detailed comparison of these data with our own is difficult because Dworkin et al., concluded their measurements at insufficiently low temperatures, obtaining only 5 points between 16 and 25°K. However, the figure shows considerable systematic difference between their results and ours below 23°K.

The most natural assumption is that the cause of this discrepancy lies in a chemical or physical difference of the specimens of CdI_2 .

Our specimen contained 0.18% of impurities; we did not detect water. In covering the range from 11°K to 300°K without gaps we found no phase transitions with accuracy to within the thermal effect of 0.5 cal./mole¹. Our calorimeter contained grains of which ~90% had linear dimensions above 1 mm and ~10% from 1 to 0.1 mm.

Dworkin et al. found a small peak in the heat capacity of their specimen at about 269-270°K, associated with a change of enthalpy of about 3 cal./mole¹. They were not inclined to ascribe this to the presence of water, but gave no other interpretation.

It is likely that this anomaly, which is associated with a small thermal effect, belongs to the category

of pseudo phase transitions. If this is the case the system under investigation is either not a single-phase system or not a single-component system. In both cases the reliability of the observed temperature dependence of the heat capacity is diminished. Therefore we do not consider the published data of Dworkin et al. [2] to be a real contradiction of our results.

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THE EFFECT OF RADIATION ON THE MOTION OF A RELATIVISTIC ELECTRON IN A MAGNETIC FIELD

A. A. Kolomenskii and A. N. Lebedev

(Presented by Academician D. V. Skobel'syn November 29, 1955)

During its motion in a magnetic field a relativistic electron is the source of intense electromagnetic radiation [1, 2]. The motion of the electron and, in particular, the radiation reaction may be investigated either classically or relativistically but, as was pointed out in [3], the results should agree so long as the following condition is satisfied:

$$E \ll E_{1/2} = mc^2 \left(\frac{R}{\Lambda} \right)^{1/2}, \quad (1)$$

where E is the energy of the electron; $\Lambda = h/mc$ is the Compton wave length; and R is the radius of the orbit.

In the papers of A. A. Sokolov and I. M. Ternov [4] attention was drawn to the role which quantum fluctuations of the radiation may play in the motion of the electron. They considered the problem in the "classical" region of Equation (1) from the point of view of quantum mechanics by solving Dirac's equation and finding the probability of transitions between stationary states of the electron in a magnetic field. The expression found in [4] for the mean square fluctuation of the orbital radius is

$$\overline{p^2} = \frac{55\sqrt{3}}{144} r_0 \Lambda \frac{\omega_0}{(1-n)^2} \gamma^2 t, \quad \gamma = \frac{E}{mc^2}, \quad (2)$$

where r_0 is the classical electron radius; ω_0 is the frequency of its orbital rotation; and n is the exponent of the magnetic field ($H \sim r^{-n}$).

In our previous work [5] we have shown that (2) and other fluctuation effects not considered in [4] may be obtained much more simply on the basis of the classical theory of betatron oscillations if in addition the statistical character of the radiation losses is taken into consideration. The validity of such a treatment of the problem in the region of (1) follows from the general theorems of the theory of radiation in quantum electrodynamics (see, for example, [6]). We note incidentally that for the usual values of the magnetic field the region of (1) extends to huge energies of the order of $E \approx 10^{15}$ ev.

In the present work we wish to draw attention to the fact that there is another important effect associated with the radiation whose character may be studied within the framework of classical concepts. This effect limits greatly the effect of the above-mentioned fluctuations.

Let us consider the motion of an electron in an instantaneous orbit lying in the plane $z = 0$. It is known that a relativistic particle radiates in a narrow cone formed by an angle $\sim 1/\gamma$ around its direction of motion at a given instant, i.e., practically along the tangent to its trajectory. Since the radiation possesses momentum a recoil force F will act on the electron in the direction opposed to its velocity and given by

$$F = \frac{W}{c}, \quad W = \frac{2}{3} \frac{e^2 c}{R^2} \gamma^4, \quad (3)$$

where W is the radiated power. We shall assume that the azimuthal component of this force, which causes a decrease of the orbital radius, is compensated.* If the electron moves at an angle to a given instantaneous trajectory, i.e., it has certain values of $\dot{\rho}$ and $\dot{z}$, then in addition to the principal azimuthal component of the force (3), radial and vertical components of the same force must act on it, which are given by

$$F_r = -\frac{W}{c^2} \dot{\rho}, \quad F_z = -\frac{W}{c^2} \dot{z}. \quad (4)$$

It is evident that F_r and F_z act as frictional forces. Their effect on the motion in the relativistic region will be considerable as they are proportional to the principal decelerating force resulting from radiation and increasing as γ^4 .

We emphasize the fact that the strong "radiation friction" which we are discussing must be distinguished from the weak radiation friction directly associated with betatron oscillations (oscillator damping). The latter effect is proportional to $\dot{\rho}$ or $\dot{z}$ and is several orders smaller than the former.

We also note that the quantum nature of the recoil may be neglected since it gives corrections of the order of only E/E_h [see (1)].

On the basis of the above, we obtain an equation for the radial betatron oscillations in the case of a constant magnetic field and (averaged) constant electron energy:

$$\ddot{\rho} + \frac{W}{E} \dot{\rho} + \omega_0^2 (1 - n) \rho = \frac{R\omega_0^3}{E} \left[\int_{t_0}^t \sum_i \epsilon_i \delta(t - t_i) dt - W(t - t_0) \right], \quad (5)$$

where $\epsilon_i = \hbar\omega_i$ is the energy of the i -th photon emitted. This equation (omitting the right hand side which takes into account the effect of quantum radiation fluctuations) can be obtained rigorously by several methods; one of these methods is based on the relativistic equation of electron motion taking into consideration the reaction force (see, for example, [1, 2]).

The betatron oscillations ρ_{bet} are described by the oscillating part of the solution of equation (5). Obtaining an expression for ρ_{bet} and averaging over the angular momenta of the emitted photons and over the emission spectrum we obtain

$$\overline{\rho_{\text{bet}}^2} = \frac{55\sqrt{3}}{96} \frac{RA}{(1-n)^2} \gamma^3 (1 - e^{-t/t_{\text{damp}}}), \quad t_{\text{bet}} = \frac{E}{W}, \quad (6)$$

where t_{damp} is the time and W is given by (3).

Equation (6) gives an expression for $\overline{\rho_{\text{bet}}^2}$ which is correct for any t and for values of E for which $t_{\text{damp}} \gg T_r$ (the period of the betatron oscillations). In the region where $T_r \gtrsim t_{\text{damp}}$ (for the usual fields this corresponds to $E \gtrsim 10^{11}$ ev), (6) must be replaced by another expression.

Equation (2) which was obtained in [4] may be derived from (6) in the limiting case $t \ll t_{\text{damp}}$ corresponding to arbitrarily small energies and arbitrarily short duration of the process.

In the other limiting case $t \gg t_{\text{damp}}$ we obtain from (6):

$$\overline{\rho_{\text{bet}}^2} = \frac{55\sqrt{3}}{96} \frac{RA}{(1-n)^2} \gamma^3, \quad (7)$$

* Such compensation of radiation losses is effected, for example, in the synchrotron through the accelerating electric field.

which differs considerably from (2) both by the absence of explicit time dependence and by much weaker energy dependence. We note that the expressions obtained in [5, 7] for oscillations of the instantaneous orbit differ from (7) only by the factor $\frac{1-n}{3-4n}$.

It is of interest that in order of magnitude (7) gives

$$\overline{\rho_{\text{bet.}}^2} \approx R^2 \left(\frac{E}{E_{1/2}} \right)^2. \quad (8)$$

This expression has a simple physical meaning: for $E \approx E_{1/2}$ the oscillations become of the same magnitude as the radius, which indicates that our method is inapplicable in this energy region. Thus from different considerations we come to our previous criterion for the validity of the classical treatment (1).

An interesting case is that in which the magnetic field and the average energy of the electron increase adiabatically with time. In this case we must add to the left hand side of (5) the term $\frac{\dot{E}}{E} \dot{\rho}$ which allows for relativistic increase of the mass. Instead of (6) we now obtain

$$\overline{\rho_{\text{bet.}}^2} = \frac{55\sqrt{3}}{144} r_0 \Lambda \frac{\omega_0}{(1-n)^2} \frac{\exp \left[-\frac{2}{3} \frac{r_0 \omega_0}{R} \int_0^t \gamma^2 d\tau \right]}{\gamma} \int_0^t \exp \left[\frac{2}{3} \frac{r_0 \omega_0}{R} \int_0^x \gamma^2 d\tau \right] \gamma^2 dx. \quad (9)$$

At large energies $\left(\int_0^t \frac{dt}{t_{\text{damp}}(t)} \gg 1 \right)$ the damping due to the increase of E is negligible

compared with the radiation friction (4), and from (9) we again obtain the characteristic expression (7). But at

small $\left(\int_0^t \frac{dt}{t_{\text{damp}}(t)} \ll 1 \right)$ (9) becomes an expression obtained in [4]:

$$\overline{\rho_{\text{bet.}}^2} = \frac{55\sqrt{3}}{144} r_0 \Lambda \frac{\omega_0}{(1-n)^2} \frac{1}{\gamma} \int_0^t \gamma^2 d\tau, \quad (10)$$

which follows directly from (5) with the friction (4) neglected but taking into account the term $\frac{\dot{E}}{E} \dot{\rho}$. Instead of by (5) the vertical oscillations (along z) will be described by

$$\ddot{z} + \frac{W}{E} \dot{z} + \omega_0^2 n z = -\frac{e}{E} \sum_i \sin \alpha_i \delta(t - t_i), \quad (11)$$

where α_i — angle of emission of the i -th photon. For the steady value of $\overline{z^2}$ instead of (7) we obtain

$$\overline{z^2} \approx \frac{\Lambda R}{n}, \quad (12)$$

which is seen to be completely independent of the energy. The physical meaning of this at first unexpected result is that although with increased energy the recoil momentum and the number of photons increase, the emission angle decreases correspondingly, and these two factors are mutually compensatory.

From the above discussion and from (6), (9), (12), etc. it follows that the radiation friction which we have considered leads to additional damping of the betatron oscillations with any compensation of the losses. As a result, of the friction effect, the oscillations due to quantum radiation fluctuations are generally of insignificant amplitude and present no danger. Thus the conclusions obtained in [4] regarding the large amplitude of this oscillation are unfounded. In particular they are not in accord with reality in the case which is of practical interest where the energy of the electron increases adiabatically.

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FRAGMENT ENERGY DISTRIBUTION FROM THE FISSION OF URANIUM AND THORIUM BY γ -RAYS

V. A. Korotkova, P. A. Cherenkov and I. V. Chuvalo

(Presented by Academician D. V. Skobel'tsyn November 21, 1955)

The fission of the uranium nucleus by neutrons [1] which was discovered in 1939 was thereafter subjected to thorough study. Among the characteristics of the fission of heavy nuclei the energy spectrum of fragments from fission by neutrons was studied. In [2-6] it was shown that the kinetic energy in this case is distributed asymmetrically among the fragments. Hence it follows that the fission of a heavy nucleus results in two fragments which differ considerably in mass. The same result is observed in the spontaneous fission of uranium [8] which was discovered by G. N. Flerov and K. A. Petrzhak [9].

Corresponding data concerning the energy spectrum of fragments from fissions due to photoexcitation were not published before 1951. In 1950-1951 we investigated [10] the energy spectra of nuclear fission fragments from a natural mixture of uranium and thorium exposed to bremsstrahlung with a maximum energy of 17.7 Mev from the synchrotron of the P. N. Lebedev Institute of the Academy of Sciences of the U.S.S.R.

For observation of fission and the determination of the fragment energies we used a differential pulse ionization chamber filled with argon. For registration of the pulses we used the electron component of the ionization pulses generated by the fission fragments in the working space of the chamber. The pulses were amplified by a proportional amplifier and recorded on film with the aid of a loop oscillograph. The electronic portion of the apparatus used in this work was similar to that used by us for the measurement of photofission cross sections [11].

The time selection scheme which we used ensured registration of only those pulses which appeared at the instant when a beam of γ -radiation passed through the ionization chamber.

In order to eliminate the dependence of the amplitude of the electron pulse on the orientation of the charged particle track, in front of the collecting electrode we installed a grid electrode. The rated efficiency of screening of the collecting electrode by this grid was 96%. At the voltages chosen for the high-potential and grid electrodes the losses of the electron component did not exceed 1% [12] and in addition reduced to a minimum the effects of recombination.

A film of the substance to be investigated was deposited on the high-potential electrode which was at a negative potential. The geometry of the chamber and the method of filling were such that the paths of the fission fragments and of the α -particles were shorter than the distance between the high-potential and the grid electrodes. In order to eliminate the distorting effect on the fragment energy spectrum of the finite thickness of the fissioning material and of edge effects, the sample was covered with a collimating grid (an aluminum plate with holes). This grid enabled us to record only those charged particles that emerged at not less than 45° to the flat surface of the sample.

The surface density of the samples used was 0.2 mg/cm² (thorium) and 0.25 mg/cm² (uranium).

To calibrate in energy units the amplitude scale of pulses recorded on the oscillograph film, we used α -particles of the samples. For this purpose, the linear amplifier provided two amplification coefficients k_α and k_f for the registration, respectively, of α -particles and fission fragments, where k_α was ten times larger than k_f . In order to ensure more reliable control of the constancy of the instrumental parameters, the calibration by α -pulses (with amplification coefficient k_α) was performed twice: before and after recording of the fission fragment pulses with amplification coefficient k_f .

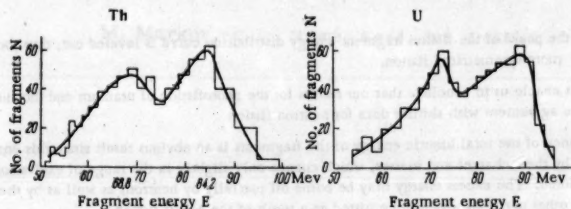


Figure 1.

The linear amplifier characteristic was obtained by photographing pulses from a special pulse generator which produced pulses similar in shape to the pulses from the ionization chamber. The amplitudes of the calibrated pulses were measured by a discriminator used in conjunction with the GSS-6 generator.

The experimentally determined fission fragment energy distribution curve was corrected for fragment energy losses in the sample and in the collimator apertures; these losses are the equivalent of losses in 3.8 mm of air under standard conditions.

Table 1
Most Probable Energy Values (in Mev)

Element	E_L	E_H	E_S
Uranium	86.5 ± 4	72.5 ± 3	159
Thorium	84.2 ± 4	68.8 ± 4	153

The final data for the number of fragments per unit energy interval for uranium and thorium are given in Figure 1. Since the photofission cross sections of the different isotopes do not differ appreciably the results obtained for natural mixtures of uranium isotopes may be ascribed to the most abundant isotope U^{238} .

Both for thorium and for uranium the photofission fragment energy distribution curves have two peaks. Consequently, just as in the case of neutron fission of the same nuclei, the fission kinetic energy is divided asymmetrically between the fragments. Hence it follows directly that in photofission the nuclei are divided predominantly into two fragments of unequal masses. The values of the most probable energies of the light (E_L) and heavy (E_H) fragments and also of their sum (E_S) which is characteristic of the most probable kinetic energy emitted in the photofission of the nuclei under investigation, are given in Table 1.

These results may be compared with data from neutron fission experiments. Although such a comparison is not strictly speaking accurate (since in neutron fission the fissioned nucleus is heavier by one unit of atomic weight than the nucleus subjected to photofission), it is entirely permissible since the results of investigations of the energy spectra of neutron fission fragments of different nuclei do not reveal any essential differences.

At first glance, our photofission fragment energy spectra differ from the analogous curves for low energy neutron fission of the same nuclei. This difference lies firstly in the smaller "valley" between the peaks and their smaller separation. In addition the value of E_L is lower and of E_H higher than the corresponding values for slow neutron fission fragments. But the total energy of the fragments E_S is in good agreement with the data for neutron fission.

Since, however, in experiments on the fission of heavy elements by neutrons of higher energies it was found that the shape of the fragment energy curve depends on the excitation energy, our data should not be compared with results obtained in experiments with slow neutrons. In fact, in the case of bremsstrahlung of peak energy 16.7 Mev which we used, with the maximum photofission cross section of uranium and thorium lying in the 13-14 Mev range, the largest contribution to the results is given by photons of the order of 12-13 Mev. This exceeds by 7-8 Mev the excitation energy of nuclei which are fissioned by slow neutrons.

There would therefore be greater justification for the comparison of our results with results obtained with neutrons of higher energies [7]. The relevant data indicate that with an increase of neutron energy,

the dip between the peaks of the fission fragment energy distribution curve is leveled out, thus showing a tendency toward more symmetrical fission.

These facts enable us to conclude that our results for the photofission of uranium and thorium are in full qualitative agreement with similar data for neutron fission.

The constancy of the total kinetic energy of the fragments is an obvious result since this energy is determined only by their charges and masses, which change only slightly in the range of excitation energy which was considered. The excess energy may be borne off partially by neutrons as well as by the fragments and, possibly, by other particles which are emitted as a result of the act of fission.

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A NOTE ON THE SYSTEMATIZATION OF HYPERONS AND HEAVY MESONS

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At the present time the new particle scheme which was proposed by Gel'man [1] is being widely discussed. Gel'man gives the charge number θ/e of a particle (of a component of a charge multiplet) as

$$\theta/e = I_z + n/2 + S/2,$$

where I_z is the component of the isotopic spin of the particle, and n is the number of particles minus the number of antiparticles (nucleons and hyperons).

Nucleons, hyperons and mesons (bosons) are provided with a "strangeness" quantum number S . For protons (p), neutrons (n) and π^0 -mesons, $S=0$; for Λ^0 and Σ , $S=-1$; for K^0 and $K^{\pm}$, $S=+1$; for a cascade hyperon Ξ^- , Ξ^0 , $S=-2$. For antiparticles, S changes sign (K^+ , $\bar{K}^0$ are also antiparticles). S is conserved in strong and electrodynamic interactions and is not conserved in weak interactions ($\Lambda^0 \rightarrow p + \pi^-$). For a change of the quantum number S , the rule is $\Delta S = \pm 1$.

Conservation of S for strong and electromagnetic interactions leads to: 1) relative stability of hyperons and heavy particles; 2) in strong interactions hyperons are produced paired with a particle whose S is equal to the S of the hyperon in magnitude but opposite in sign.

It is the purpose of the present note to show that the author's discussions [1, 2] of the possible nature of hyperons and K-mesons lead to a scheme that is very similar to that of Gel'man and that the quantum number S can be given a physical interpretation. We consider hyperons to be excited states of nucleons. An oscillator model* is suggested to illustrate these excitations mathematically. The zero-th level of excitation of the nucleon oscillator corresponds to a neutron or proton (n_0, p_0); the first level of excitation corresponds to Λ^0 or Σ (i.e., n_1, p_1); the second level corresponds to a cascade hyperon Ξ ($n_2, \dots$); in principle it is possible to have a hyperon of the S -th level of excitation (n_S, p_S).

As will be seen below the number of an excited state of a hyperon agrees with the number mentioned above ($S=0, 1, 2$). The selection rule $\Delta S = \pm 1$ is a rule only for transitions between neighboring excitations.

Since in the matrix element for transitions between states of different excitation the essential factor is $(kr_0)^m$ where k is the wave number of the π -mesons and r_0 represents the dimensions of the nucleon oscillator, by choosing a small r_0 it is possible to obtain the observed lifetime of hyperons. Smallness of r_0 ensures relative stability of the hyperons, i.e., it ensures conservation of S , the excitation number of the hyperons "for strong and electrodynamic interactions".

But this model allows another mechanism for the creation of excited nucleons, namely the creation of

* It is significant that the physical results discussed below are characteristic of the entire class of equations that can describe excited states of particles.

a pair of nucleons in excited states.

For example,

$$\pi^- + p_0 \rightarrow p_0 + p_{-S} + n_S, \quad (1)$$

where S is the excitation number of the nucleon; and $-S$ designates the same excitation of antinucleon. This process is also possible in Gel'man's system since, according to Gel'man, n_S and p_S possess opposite "strangeness" S and the process (1) occurs with conservation of the "strangeness" S .

We have also suggested that nucleons and antinucleons may be united as bosons*, for example:

$$\pi^0 \equiv \pi_{0,0} \equiv (p_0 + p_{-0}); \quad \pi_{0,-1} \equiv (p_0 + p_{-1}) \equiv \theta^0; \quad \pi_{s,-S} \equiv (p_s + p_{-S}). \quad (2)$$

It is important to note that if a number S is assigned to an excited nucleon and $-S$ to an antihyperon (or vice versa), in our scheme we thereby determine the "strangeness" S for a boson $\pi_{0,-S}$; it agrees with the strangeness of the excited antihyperon.

For example, the reaction

$$\pi^- + p_0 \rightarrow p_0 + p_{-1} + n_1$$

$\downarrow \quad \quad \downarrow$
 $\bar{\theta}^0 + \Lambda^0$

(3)

is permitted by Gel'man's formulation, since the "strangeness" quantum numbers for θ^0 and Λ^0 are equal in magnitude and opposite in sign.

Correspondingly for θ^- we have: $\theta^- \equiv (n_0 + p_{-1})$. As in Gel'man's scheme, for θ^0 and θ^- there are obtained here the antiparticles:

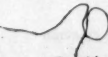
$$\bar{\theta}^0 \equiv (p_{-0} + p_1); \quad \theta^+ \equiv (n_{-0} + p_1). \quad (4)$$

But for θ^- and θ^+ there are other possibilities:

$$\begin{aligned} {}_1\theta^- &\equiv (p_0 + n_1), \quad S = -1; & {}_2\theta^- &\equiv (n_0 + p_{-1}), \quad S = 1; \\ {}_1\theta^+ &\equiv (n_{-0} + p_1), \quad S = -1; & \theta^+ &\equiv (p_0 + n_1), \quad S = 1. \end{aligned} \quad (5)$$

${}_1\theta^-$ and ${}_2\theta^-$ differ physically in that the process ${}_1\theta^- + p_0 \rightarrow \Lambda^0 + \pi^0$ is permitted but ${}_2\theta^- + p_0 \rightarrow \Lambda^0 + \pi^0$ is forbidden. It must be emphasized that ${}_1\theta^0$ may be produced only together with another K-meson, for example,

$$\pi^- + p_0 \rightarrow p_0 + (n_1 + p_{-1}) + (p_{-0} + p_0) \rightarrow p_0 + {}_1\theta^- + \theta^0. \quad (6)$$



Considering a cascade hyperon Ξ^- to be a nucleon of the next excitation number we designate it by p_{-2}^- and its positive antiparticle by p_{-2}^+ . A pair of such hyperons could be created in the reaction

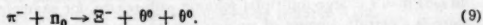
*For example, in the sense of a generalization of the familiar idea of Fermi and Yang. A similar suggestion was made by M. Levy and R. Marshak [3].



Of course, it is impossible to exclude the existence of a cascade hyperon Ξ^0 :



It must be noted that Gel'man in this case does not introduce a new particle θ_{-1}^0 with strangeness $S = 2$, but composes this number of the S numbers of the two θ^0 -particles:



Such a reaction is forbidden in our scheme if θ_{-1}^0 does not decay rapidly into two θ^0 particles.

The process $\Xi^- + p_0 \rightarrow \Lambda^0 + \Lambda^0$ in our scheme unlike that of Gel'man's is not "fast"; it can only take place at all because the cascade hyperon captured by the proton remains in the vicinity of the proton for a long time; this process is in competition with the slow reaction $\Xi^- + p_0 \rightarrow p_0 + \Lambda^0 + \pi^-$ which can leave a hyperon bound in a nuclear fragment. The same can be said for the slow reaction $\Xi^- + p_0 \rightarrow \Lambda^0 + \pi^0$.

Table 1

No.	Reaction	Pair scheme	According to Gel'man
1	$\pi^- + p_0 \rightarrow p_0 + p_{-1} + n_1$ $\downarrow \theta_{-1}^0 + \Lambda^0$	Allowed	Allowed
2	$\pi^- + p_0 \rightarrow \Lambda^0 + \pi^0$	Forbidden	Forbidden
3	$\pi^+ + p_0 \rightarrow p_0 + n_{-1} + p_1$ $\downarrow \theta_{-1}^+ + \Sigma^+$		
4	$\pi^+ + n_0 \rightarrow n_0 + n_{-1} + p_1$ $\downarrow \theta_{-1}^0 + \Sigma^+$	Allowed	Allowed
5	$\Xi^0 + p_0 \rightarrow (p_{-0} + n_1) + p_0 \rightarrow (p_{-0} + p_0) + n_1$ $\downarrow \pi^0 + \Lambda^0$	.	.
6	$\pi^- + n_0 \rightarrow n_0 + p_{-1} + n_1$ $\downarrow \Xi^0 + \Lambda^0$	.	.
7	$\pi^- + n_0 \rightarrow n_0 + p_{-1} + p_1$ $\downarrow \Xi^0 + \Sigma^+$	.	Forbidden
8	$\pi^- + p_0 \rightarrow p_0 + (n_1 + p_{-1}) + (p_0 + p_{-1}) \rightarrow p_0 +$ $+ \Xi^0 + \theta^0$	.	Allowed
9	$n_0 + n_0 \rightarrow \Lambda^0 + \Lambda^0$	Forbidden	Forbidden
10	$\Xi^0 + p_0 \rightarrow \Lambda^0 + \pi^0$	.	Allowed
11	$\pi^- + n_0 \rightarrow \Xi^- + \theta^0 + \theta^0$	.	.
12	$\pi^- + n \rightarrow \theta_{-1}^+ + \Xi^-$	Allowed	.
13	$\Xi^0 + p_0 \rightarrow \Sigma^+ + \pi^-$	.	.
14	$\Xi^0 + n_0 \rightarrow \Lambda^0 + \pi^-$	.	.
15	$\theta^+ + p_0 \rightarrow \theta^0 + p_0 + \pi^+$	.	.
16	$\theta^+ + n_0 \rightarrow \theta_0 + p_0$	.	.

whereas the reaction $\Lambda^0 + p_0 \rightarrow \Lambda^0 + \pi^0$ is fast.

The process $n + n \rightarrow \Lambda^0 + \Lambda^0$ is forbidden by both schemes (for the latter process B. Pontecorvo has obtained the experimental result $\sigma < 10^{-31} \text{ cm}^2$).

Possible elastic scattering of heavy mesons by nucleons is given by

$$\theta^+ + p'_0 \rightarrow (n_{-1} + p_0) + p'_0 \rightarrow (n_{-1} + p'_0) + p_0 \rightarrow \theta^+ + p_0.$$

$$\theta^+ + n_0 \rightarrow (n_{-1} + p_0) + n_0 \rightarrow (n_{-1} + n_0) + p_0 \rightarrow \theta^0 + p_0.$$

These processes may be accompanied by the emission of bremsstrahlung π -mesons since we are talking essentially about the collisions of nucleons with antinucleons and hyperons, interacting through a π -meson field.

The collisions of θ^+ and θ^- with nuclear matter at low energies can only induce slight changes in the energy of the system exclusively at the expense of their kinetic energy. On the other hand, other reactions (for example Λ^0) lead to much more considerable changes in the interacting systems.

In Gel'man's charge multiplet scheme with regard, for example, to the prediction of a Σ^0 particle, etc., there is a certain amount of ambiguity (Λ^0 could be replaced by Σ^0). Thus in our interpretation, the conservation of the special number S coincides with the conservation of the nucleon excitation number: $S = 0, 1, 2$. In other words, the conservation of S denotes the conservation of the number of nucleons separately for each excitation, but in our scheme this is not an absolute rule, since it can be broken at very high energies or for long times.

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RELATIVE CAPTURE AND RECOMBINATION CROSS SECTIONS OF OPTICAL ELECTRONS IN ZnS-Cu, Co PHOSPHORS

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In the kinetics of phosphorescence an important part is played by the cross sections of the capture of free electrons σ and of their recombination σ_0 with ionized luminescence centers. It is necessary here to distinguish thermal and optical electrons (electrons may be freed from a trap by either heat or light), since it was shown in [1, 2] that they act differently because of the difference of their energies. In [3] we calculated σ_0 and σ for thermal electrons on the basis of an analysis of thermal emission curves of the phosphor following excitation of short duration. In the present paper we evaluate σ_0/σ for thermal electrons from natural decay curves and for optical electrons from curves of decay induced by infrared light.

We confine ourselves to a phosphor containing only one kind of capture level since we can thus avoid complications resulting from the distribution of electrons on levels of different depths.

We first consider the case in which the localized electrons are liberated optically. Investigations of decay in this instance must be conducted at such a low temperature that thermal liberation of the trapped electrons would be practically impossible. Under infrared irradiation, the kinetics of luminescence follows the scheme of a bimolecular reaction [4].

If the following conditions are satisfied: 1) there is no external quenching; and 2) only an insignificant fraction of the levels are filled with electrons, the following differential equation holds:

$$\left(1 + \frac{\sigma\nu}{\sigma_0 n}\right) \frac{dn}{dt} = -\alpha En, \quad (1)$$

where n is the concentration of ionized luminescence centers; ν is the concentration of trapping levels; E is the intensity of infrared light; and α is a certain constant. The quantity αE designates the probability of optical liberation of localized electrons by infrared light.

By integrating (1) we find that

$$\ln \left(\frac{n}{n_0} \right) - \frac{1}{n_0} \left(\frac{n_0}{n} - 1 \right) = -\alpha Et, \quad (2)$$

where n_0 is the value of n for $t = 0$ and

$$\frac{1}{n_0} = \frac{\sigma_0 n_0}{\sigma\nu}. \quad (3)$$

Thus from the shape of the decay curve, the ratio σ_0/σ can be evaluated if we know another ratio, n_0/ν .

For the experiments we chose ZnS-Cu(10^{-4}), Co($6 \cdot 10^{-6}$) with a CaCl_2 (5%) flux prepared in a nitrogen atmosphere at 1100° . For this phosphor the thermal emission curve has two clearly separated peaks at around -140 and 65° , thus indicating two different kinds of capture levels. At first glance it might appear that this phosphor is unsuitable for our purposes because of shallowness of the capture levels. But on the basis of subsequent facts it may be assumed that the effect of the shallow levels is negligibly small if prior to irradiation

with infrared light there are no electrons on these levels. It was found experimentally that the capacity of a wave length $\sim 0.9 \mu$ to liberate electrons from shallow levels is greater than for deep levels and the probability of localization of optical electrons on shallow levels is many times smaller than on deep levels.

The experiment was conducted as follows. The phosphor was excited for a prolonged period at room temperature. For 45 seconds after termination of the excitation it was cooled at the temperature of liquid nitrogen. Then the phosphor was irradiated with $\sim 0.9 \mu$ at that temperature for 10, 15, etc., seconds and the corresponding afterglow light sums $\bar{n}$ were measured by the method of thermal emission (in all cases the light sum under a low temperature peak was $\sim 10^4$ times smaller than under a high temperature peak so that the effect of shallow levels could indeed be neglected).

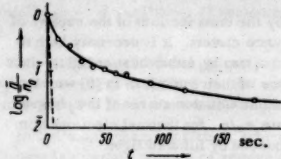


Figure 1. Dependence of $\log \frac{\bar{n}}{n_0}$ on t , the duration of irradiation with infrared light $\lambda \sim 0.9 \mu$.

The values obtained for $\bar{n}$ fit well on curve (2), whose parameters $\bar{n}_0$ and αE were determined from the data of two measurements (see Figure 1). It should be noted that $\bar{n}_0$ can be determined with sufficient accuracy only when $\bar{n}_0 \sim 1$.

ν was determined from the concentration of Co since the deep levels are created by cobalt [5]. n_0/ν was evaluated for a thin layer of the phosphor by the method described in [3]. It was shown that $n_0/\nu \sim 0.16$, $\bar{n}_0 \sim 0.22$. Then from (3) it follows that

$$\frac{\sigma_0}{\sigma} \sim 2. \quad (4)$$

For thermal electrons σ_0/σ can be determined analogously. In (2), αE must be replaced by a quantity representing the probability of thermal liberation of a localized electron:

$$p = \frac{1}{\tau_0} e^{-\epsilon/kT}, \quad (5)$$

where ϵ is the localization energy of the electron; T is the absolute temperature; k is the Boltzmann constant, and τ_0 is a constant.

The experiment was conducted at 20, 65 and 90°. The results showed that at 20 and 90° the shape of the decay curve of $\bar{n}$ with time differs markedly from the theoretical curve. At 20° there is evidently still a certain amount of nonuniformity of the deep levels. In fact, since the probability of liberation of an electron according to (5) depends exponentially on ϵ , a small change of ϵ can strongly affect p . At 90° the decay curve resembles an exponential curve which indicates the appearance [6] of a new factor—external quenching, a subsequent first order reaction. Good agreement with experiment was obtained at 65° (see Figure 2) in the absence of external quenching and when the shallower levels cease to have any effect (there is a peak of the thermal emission curve at just 65°). By this method, just as in the case of optical electrons, it was found that

$$\frac{\sigma_0}{\sigma} \sim 25. \quad (6)$$

We had found previously by another method [3] that at -140° for thermal electrons in the same phosphor

$$\frac{\sigma_0}{\sigma} \sim 200. \quad (7)$$

This difference can be explained as follows. According to the diffusion theory of phosphorescence [1] which is confirmed by [3], the recombination cross section is

$$\sigma_0 \sim \frac{1}{T^2}. \quad (8)$$

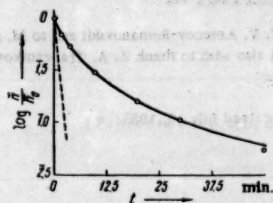


Figure 2. Dependence of $\log \frac{n}{n_0}$ on the time of decay t at 65° .

A neutral capture center corresponds at best (with regard to electron trapping) to a dipole created by the approach of the electron. In the field of the dipole the potential energy of the electron at a distance r from the center of the trap [7] is

$$V \sim \frac{1}{r^2}. \quad (9)$$

The effective trapping radius r_T is determined by the relation (the equality of potential and mean kinetic energy of the electron [1])

$$V = \frac{3}{2} kT. \quad (10)$$

and then from (9) and (10) it follows that

$$\sigma \sim \pi r_T^2 \sim \frac{1}{VT}. \quad (11)$$

Consequently σ is generally only weakly dependent on T .

From consideration of the temperature dependence of σ_0 and σ , instead of $\sigma_0/\sigma \sim 25$ at $T = T_1 = 65^\circ$, we find using (8) and (11) that at $T = T_2 = -140^\circ$,

$$\frac{\sigma_0}{\sigma} \sim \left(\frac{T_1}{T_2} \right)^{1/2} \sim 100; \quad (12)$$

this is in much better agreement with the value of ~ 200 obtained by another method.

The following may be said concerning the conditions under which the experiments were conducted. The agreement between the experimental decay curves and the theoretical curve which was calculated for an infinitely thin layer indicates that the effect of the thickness of the phosphor is small although it was practically infinitely thick in relation to the absorption of light. This is evidently explainable by the fact that although with penetration into the phosphor the intensity of the exciting light is strongly attenuated, yet due to the simultaneous attenuation of induced emission, at full excitation, the concentration of ionized centers may no longer depend in a large range on the depth of penetration of the exciting light [8]. Therefore the main part of the excited layer of the phosphor contains a constant concentration of ionized centers, i.e., it is uniformly excited.

Thus we have established the difference between the ratios σ_0/σ for thermal and optical electrons. The new result for thermal electrons agrees with the value based on the diffusion theory, and the result for optical electrons agrees with the data of L. I. Ankina [9] and apparently also with the data of other authors [10].

The calculated results make it possible to judge as to the validity of one theory of phosphorescence or another. For example, the assumption in [11] that $\sigma_0/\sigma < 1$ can hardly be considered realistic. The statement that in the case of ZnS-Cu, Co for thermal electrons σ_0 is independent of T and $\sigma \sim e^{-W/kt}$ (where W is the thermal activation energy required to attach an electron to a trap) in [12] does not agree with our results.

It should be noted that although according to calculation the phosphor should possess levels of only one kind it appears from experiment that the method is applicable to a more complicated case. It is only required that to determine σ_0/σ for optical electrons on shallow levels the electrons should not, or almost not be localized, and that to determine σ_0/σ for thermal electrons the deepest levels should be uniform.

The method is also apparently applicable when the levels are pretty well filled. In this case according to [11] and [13], Equation (2) becomes more complicated.

In conclusion I wish to acknowledge my deep indebtedness to V. V. Antonov-Romanovskii and to M. A. Konstantinova-Shlezinger for guidance and interest in the work, and I also wish to thank Z. A. Trapeznikova and V. V. Shchaenko for preparing the phosphors.

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IN RAYLEIGH SCATTERING OF LIGHT

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1. The fine structure of a Rayleigh scattering line which was predicted by L. I. Mandelshtam [1] and by Brillouin [2] and observed in liquids by E. F. Gross [3] is a triplet. The relative intensities of the components of the triplet according to L. D. Landau and G. Plachek [4] is given by

$$\frac{I_C}{2I_{M-B}} = \gamma - 1, \quad (1)$$

where I_C is the integral intensity of the central-component of the triplet; $2I_{M-B}$ is the integral intensity of the shifted components; and γ is the ratio of the specific heats c_p/c_v . The Landau-Plachek [4] equation has frequently been confirmed experimentally on various objects.

From the most extensive experimental studies of this problem [5, 6] it was concluded that (1) disagrees with experiment in all cases, whereas in other work [7, 8] it was found that (1) is in agreement with experiment.

An analysis of the experimental methods used to determine the relative intensities in a Rayleigh triplet shows that these methods do not enable us to obtain reliable experimental data for a comparison of theory with experiment. On the other hand, the derivation of (1) involved the neglect of factors which may sometimes be significant. The present work aims at improving the theory and obtaining reliable experimental data.

2. In our previous paper [9] based on the general theorems of statistical thermodynamics and neglecting no factors at all we found that for the integral intensity of light scattered by adiabatic density fluctuations (Mandelshtam-Brillouin components):

$$I_{M-B} \sim \left(\frac{\partial \epsilon}{\partial \rho} \right)_T^2 \beta_s k T \rho, \quad (2)$$

and for the integral intensity of light scattered by isobaric density fluctuations (central component):

$$I_C \sim \left(\frac{1}{\alpha} \frac{\partial \epsilon}{\partial T} \right)_P^2 \frac{\alpha^2 k T^2}{c_p}, \quad (3)$$

whence the ratio of the intensities of the central component of the triplet and the side components is given by

$$\frac{I_C}{2I_{M-B}} = \frac{\left(\frac{1}{\alpha} \frac{\partial \epsilon}{\partial T} \right)_P^2}{\left(\frac{\partial \epsilon}{\partial \rho} \right)_T^2} \frac{\alpha^2 T}{c_p \rho \beta_s}, \quad (4)$$

where $\epsilon \equiv n^2$ (n is the index of refraction); ρ is the density; α is the coefficient of expansion; c_p is the specific heat at constant pressure; β_s is the adiabatic compressibility and T is the absolute temperature.

(4) is reduced to (1) if we neglect $\left(\frac{\partial \epsilon}{\partial T}\right)_p \frac{\alpha T}{c_p \rho \beta_s}$ (in this case $\left(\frac{1}{\alpha} \frac{\partial \alpha}{\partial T}\right)_p = 1$) and keep in mind

the well-known thermodynamic relations $\frac{\alpha^2 T}{c_p \rho} = \beta_T - \beta_s$; $\beta_T / \beta_s = \gamma$. In previous work [10-12] we showed that β_s and $\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_s$ depend essentially on the frequency of the elastic wave. In this case, the thermodynamic equations (2-4) cannot be applied to our problem since dispersion leads to irreversible processes and a rigorous examination of the problem including dispersion has still not been achieved.

However, for the intensity of light scattered by adiabatic density fluctuations it is possible to obtain an expression by considering the diffraction of light by an elastic thermal wave. In this case we obtain an expression which agrees formally with (2), but β_s and $\left(\rho \frac{\partial \epsilon}{\partial \rho}\right)_s$ must be taken as corresponding to the frequency of the elastic wave, i.e., $\sim 10^{10}$ cycles. In the expression for the intensity of the central component (3), it is possible to stop in first approximation with the static values of the included quantities, since entropy fluctuations are dissipated extremely slowly compared with adiabatic density fluctuations. Indeed, entropy fluctuations lead to an unshifted line in the spectrum of scattered light, with a half breadth of 10^7 c. The relaxation time which determines c_p , obtained on the basis of measurements of the dispersion of sound velocity [10, 11] is $\tau \approx 10^{-9}$ sec., so that c_p may be taken as constant. A certain amount of error may be introduced by $\left(\frac{1}{\alpha} \frac{\partial \alpha}{\partial T}\right)_p$ since a part of this which depends on $\rho \frac{\partial \epsilon}{\partial \rho}$ as was shown in [12] may change somewhat at frequencies $\sim 10^7$ c. But from the data in [12] it appears that a large error must not be anticipated. It thus follows that the calculation of $I_{C/21M-B}$ must be carried out according to (4) but that the result of this calculation may differ very greatly from the calculation by (1) in cases where a considerable dispersion of the velocity of sound is observed.

3. The shortcomings of previous experimental investigations of the relative intensities in the Rayleigh triplet have been discussed in part in the literature [13, 14]. However, it must be pointed out that the most essential methodological defect of all the earlier investigations is that the "background line" on the photometric curves of the triplet was drawn arbitrarily.

Because of the finite breadth of the excited line and the closeness of the components to each other, the fine structure components are always more or less superimposed; in addition, there is superimposed on the

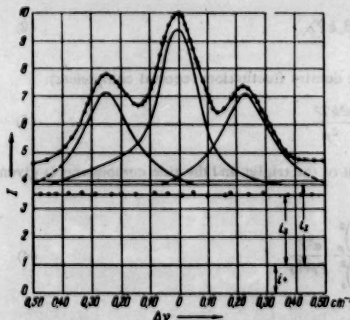


Figure 1. Intensity distribution in I_x and I_z of the components of the Rayleigh scattered line in benzene.

Rayleigh triplet the lamp background and the "continuous spectrum" of great intensity due to anisotropic fluctuations. Thus the Rayleigh triplet is seen against a more or less intense continuous spectrum.

In all previous work the "background line" from which the intensities of the triplet components must be calculated was drawn "by rule of thumb" and was not free of arbitrariness. Thus the study of the same spectrograms can easily lead to very different results. Our method enables us to obtain fine structure photographs simultaneously in the two mutually perpendicular directions of polarization Z and X (see Fig. 1). (In the Z-component the vector of the electric field E is perpendicular to the plane of scattering). From this photograph we can draw an unambiguous "background line" and obviate to a large degree the other shortcomings of earlier work.

* The author is indebted to V. L. Ginzburg for this comment.

Table 1
Relative Intensities of the fine structure components

Substance	$I_{\text{C}}/I_{\text{M}} - n$		
	Measured	Calculated	
		By Eq. (1)	By Eq. (2)
Benzene	0.08 ± 0.25	0.46	0.70
Acetone	0.59 ± 0.12	0.40	0.57
Carbon disulfide	0.93 ± 0.18	0.56	0.98
Carbon tetrachloride	1.10 ± 0.30	0.46	0.84
Water	~ 0	0.0068	0.0115

In addition to the profile of the triplet, the profile of the exciting line (Hg λ 4358Å) is drawn, from which we determine the portion of the light intensity which is to be ascribed to the continuous spectrum of the source. Hence it is possible to determine I_0 in the Z-component of the scattered line (Figure 1).

In an independent experiment it was found that the depolarization coefficient of the "continuous background" induced by scattering from anisotropic fluctuations is $6/7$ and is independent of the frequency, in agreement with theory [15, 16]. This result makes it possible by determining I_X to find the fraction of light scattered by anisotropic fluctuations in the Z-component I_Z (Figure 1).

Between the horizontal portion of the "tail" of the visible profile of the Z-component and the line corresponding to I_Z there is a small amount of extra intensity due to superimposition of the "tails" of adjoining components of neighboring interference orders, so that it is possible to determine the intensity belonging to a single order and by the method of successive approximations [17] to separate from the overall contour all three components of the fine structure. In Figure 1 these are shown by thin lines. The integral intensity of the components is determined by planimetry. The complications of the photometric procedure for the deciphering of the components leads to an error of 20-30%.

4. The results of the measurements of the relative intensities are given in Table 1. The values for the Rayleigh triplet as calculated from (1) and (4) are also given. Considering the appreciable experimental errors and the approximate character of (4) the agreement between the calculated and experimental values must be deemed good. Thus, in general outline, the problem of the relative intensities of the fine structure components may be considered solved both experimentally and theoretically.

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A STATISTICAL ESTIMATE OF THE ENERGY OF PARTICLES WHICH GENERATE MIXED SHOWERS

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(Presented by Academician D. V. Skobeftsyn November 11, 1955)

I. One of the most important factors involved in a quantitative study of high energy collisions that give rise to electron-nuclear ("mixed") showers (to be designated hereafter as ENS) is the energy ϵ of the primary particle. However, up to the present, there has been no objective method of estimating this quantity for all showers. The usual analysis of the angular distribution of the shower particles gives only the velocity of the center-of-mass. The latter in turn is not uniquely related to the energy ϵ of the primary particle but also depends [1-3] on the number ν of nucleons in the nuclear target nucleus that participate in the collision. On the other hand an estimate of ϵ based on an analysis of the soft component of the ENS depends on definite assumptions concerning the magnitude of the ratio $\pi^0/\pi^\pm$ and also of the ratio of the number of pions to the number of all the shower particles. These difficulties in the interpretation of a single collision have induced us to develop a statistical approach to the problem.

We shall assume that the detector of the ENS, — for example, a row of shower counters with a given coincidence scheme or a shielded ionization chamber, — will detect any incident (strongly interacting) particle whose energy exceeds a certain "threshold" value ϵ_0 . If the (normalized) spectrum $F(\epsilon)d\epsilon$ of the primary particles is given, the mean energy will be calculated by

$$\langle \epsilon \rangle = \int_{\epsilon_0}^{\infty} \epsilon E(\epsilon) d\epsilon. \quad (1)$$

Thus ϵ_0 may serve as an estimate of the mean energy of the particles that trigger the detector for a given combination of counters or thresholds of the apparatus used to detect the ENS. We shall describe below the method of determining ϵ_0 based on the energy dependence of the rate of decay** of the pions in the high-energy N-component of the cosmic radiation, as well as some preliminary results obtained by this method.

II. Let N_1 be the number of counts per time unit from the ENS at height z_1 ; N_2 at height z_2 ($z_2 < z_1$) and N'_1 at height z_1 under a thickness of absorber equivalent to nuclear absorption of the generating particles in an air layer of thickness X (g · cm⁻²) between z_1 and z_2 . If the (density) transition effect [4] is properly eliminated, the difference between N'_1 and N_2 may be ascribed only to the spontaneous decay in flight of at least some portion of the generating particles. We shall assume at first that the generating component consists only of pions (of lifetime τ_0). We define the compensation effect Q_0 as the ratio

$$Q_0 \equiv \frac{N'_1}{N_2} = \left(\int_{\epsilon_0}^{\infty} F(\epsilon) d\epsilon \right) / \left(\int_{\epsilon_0}^{\infty} F(\epsilon) \exp \left[-\mu \frac{z_1 - z_2}{\tau_0 \epsilon} \right] d\epsilon \right) \quad (2)$$

(considering only the cases for which $\epsilon \gg \mu$, i.e., $\beta \approx 1$). For the energy spectrum

$$F(\epsilon) d\epsilon \sim e^{-(1+s)\epsilon} d\epsilon \quad (3)$$

* German spelling: E. M. Friedländer.

** Since for energies $\epsilon \gg \mu$, where μ is the rest mass of the pion ($h = c = M_p = 1$), the value of ϵ_0 is nearly identical for nucleons and pions.

it follows from (2) that

$$Q_0^{-1} = e_1^{-1} {}_1F_1(1, 1 + \alpha, t_0), \quad (4)$$

where $t_0 = \mu \frac{z_1 - z_2}{t_0 \epsilon_0}$ and ${}_1F_1$ is the degenerate hypergeometric function which may be approximated with sufficient accuracy (in the region which concerns us) by an exponential: thus

$$Q_0 \approx e^{\chi}, \quad (5)$$

where χ depends on α and on the angular distribution of the primary particles (for $\alpha = 2$ and angular distribution $\sim \cos^2 \theta$, $\chi \sim 1$).

Now the generating component consists of nucleons and mesons. We shall denote by ρ the ratio of the number of nucleons to the number of mesons at height z_1 . Then the observed compensation effect Q will be:

$$Q = (1 + \rho) / (Q_0^{-1} + \rho). \quad (6)$$

By measuring Q for two or more values of t_0 (that is, by recording the ENS at three or more heights) we can calculate ϵ_0 and ρ by one of the methods described in [5, 6] (for example, for sufficiently large $z_1 - z_2$, Q approaches saturation $Q_\infty = (1 + \rho) / \rho$ and any intermediate value of $z_1 - z_2$ then gives ϵ_0). It must be noted that this method also gives the ratio of the number of nucleons to the number of mesons in an energy region that is inaccessible to ionization instruments.

III. Preliminary measurements taken by the method described above were published in the summer of 1954 by a group of workers of the Physics Institute of the Academy of Sciences of the Rumanian People's Republic (Fridlender, Iusim and Maier) [6] for various heights up to 2500 m in the Bucegi mountains.

The detector of the ENS was a set of shower counters arranged for triple coincidences and shielded by a 10 cm layer of lead. To eliminate the transition effect, a measurement was taken of the absorption thickness Λ_C in graphite under an additional 40 cm thickness of lead. This gave $\Lambda_C = (270 \pm 20) \text{ g} \cdot \text{cm}^{-2}$. The values of Q were calculated from the equation

$$N'_1 = N_1 e^{-\chi / \Lambda_C} \quad (7)$$

From ten values obtained for Q the method of least squares was used to determine ϵ_0 and ρ , giving $\epsilon_0 = (12.4 \pm 3.6) \text{ Bev}$, and $\rho = 0.45 \pm 0.20$. Thus our ENS detector was triggered by particles with a mean energy of $\langle \epsilon \rangle = (24.8 \pm 7.2) \text{ Bev}$.

These results may be checked as follows. It is known [7] that the absorption thickness Λ_C of the incident particles is related to the interaction length Λ_{0C} by

$$\Lambda_{0C} = \Lambda_C (1 - s), \quad (8)$$

where s is the average number of N-particles of the kind recorded in our experiment. For $\Lambda_{0C} = 70 \text{ g} \cdot \text{cm}^{-2}$ this gives $s \sim 0.8$, which is in good agreement with the value for protons of the same energy obtained in [8] on the basis of geomagnetic effects.

The self-consistency of our results was checked as follows; it is easily seen that Q_0 is related to Q and Q_∞ by

$$Q_0 = \frac{Q(Q_\infty - 1)}{Q_\infty - Q} \quad (9)$$

On the other hand from (4) and (6) we have

$$Q_0 \approx \xi \equiv e^{x/\xi}, \quad (10)$$

whence

$$\ln \xi = \chi \frac{x_1 - x_2}{x_0 x_0} \quad (11)$$

A straight line drawn in logarithmic coordinates through the experimental points (by the method of least squares) passed through the origin within the limits of statistical errors.

IV. The results obtained by our method may be used to study high-energy nuclear interactions in the following two ways:

- A detector calibrated for "average energies" may be used to trigger a Wilson cloud chamber or a hodoscopic set of counters; thus it is possible to carry out a statistical study of an ENS with known average energy;
- The values of $\langle \epsilon \rangle$ and ρ may be used under definite assumptions regarding a nucleon cascade in air, for the checking of different theories of the creation of mesons in collisions of nucleons with nuclei. In this section we shall give an example of the possibilities involved in the use of the second variant of this method.

Let us assume that the particles recorded by our ENS detector originated in another ENS generated in air by the collision of a particle of very high energy E (compared with $\langle \epsilon \rangle$) with an air nucleus. This collision can be described* [1] as the interaction of a primary particle with ν nucleons of the target nucleus which lie in its path in the nuclear matter ($\langle \nu \rangle = A^{1/3}$). Let γ_c be the relativistic contraction factor $\gamma_c = (1 - \beta^2)^{-1/2}$ and E_c the total energy of $\nu + 1$ nucleons, in the center-of-mass system. If K is the energy dissipated in the creation of mesons the multiplicity of the "parent" shower can be given in general as

$$n = B (KEc)^{\omega} \gamma_c^{-2\omega}, \quad (12)$$

where the parameters B , ω , and δ are quantities characterizing different theories of meson production.**

It was shown in [1] that $E_c \approx (2\nu E)^{1/2}$ and in [9] that $\gamma_c \approx (E/2\nu)^{1/2}$. Setting $g = \delta - \omega$ it is easily shown that

$$n \sim \sqrt{g+2\omega} E^g, \quad (13)$$

$$\rho^* \sim \frac{\nu+1}{\sqrt{g+2\omega}} K^{-\frac{g+2\omega}{1-g}} \langle \epsilon \rangle_{\pi}^{-\frac{g}{1-g}}, \quad (14)$$

where $\langle \epsilon \rangle_{\pi}$ is the average pion energy, and ρ^* is the ratio of the number of nucleons to the number of pions in the N -component of the "parent" shower.

The nucleons recorded in our experiments together with the pions of the same average energy come from a "parent" shower of energy

$$E' = \sigma E, \quad (15)$$

where the quantity

$$\sigma = \frac{1}{\rho^*} \frac{K}{1-K}, \quad (16)$$

* The author reported to the Physical Institute on November 10, 1953 concerning this model of the collision of a nucleon with a nucleus. Similar considerations were developed independently by Cocconelli [2] and Haber-Schaim [3].

** Only multiple production is considered here.

is generally not equal to unity

For the energy spectrum (3) ρ^* is related to the observed ratio ρ by the relationship

$$\rho^*(\alpha-1) \rho = \left[\frac{3(1-K)}{2K} \right]^\alpha$$

Equations (14) and (16) can be solved with respect to K and ρ^* . Then (16) gives σ and we can easily obtain the average energy E :

$$E = \frac{(v+1) K \rho \langle \epsilon \rangle_\pi}{3(1-K)^2} \quad (17)$$

As typical examples we shall consider the theories of Heisenberg [10] ($\delta = \frac{1}{2}$, $\omega = 0$, $g = \frac{1}{2}$) and Fermi-Landau [11, 14] ($\delta = \frac{3}{8}$, $\omega = \frac{1}{8}$, $g = \frac{1}{4}$). Taking $\alpha = 2$ [12], $\langle v \rangle = 2.4$ for air and using our data for ρ and $\langle \epsilon \rangle_\pi$ and also assuming that only particles moving in the center-of-mass system in the forward-directed cone are effective in the sense of generating an ENS in the laboratory system of coordinates, and that π^+ -mesons comprise one-third of all the pions generated, we obtain the values given in Table 1.

Table 1

	$\omega=0, g=1/2$	$\omega=1/8, g=1/4$
K	>0.98	~ 0.7
ρ^*	$\leq 2 \cdot 10^{-3}$	~ 0.7
σ	≤ 0.07	~ 1.2
E	~ 2.10	~ 130
n	>250	~ 7.5

The values in the first column of Table 1 are in sharp disagreement with all known experimental data. Thus n is ten times as large as the multiplicity observed by the Rochester group [9] for the same energies. The analysis of nucleon cascades in extensive atmospheric showers by the Pamir group [13] showed that it is probable that $\sigma \sim 1$. Finally, by taking for K the value given by Heisenberg: $K \sim e^{-\mu b}$, we find that for the average value of the collision parameter b given by $b_{1/2} = 1/\sqrt{2} \mu$, K should be only about 0.5.

On the other hand, all values in the second column of Table 1 (Fermi-Landau theory) are in good agreement with experimental data (see, for example [1], where the results of a detailed analysis of the data from bursts offers strong arguments in favor of that theory).

Conclusion. The preliminary results thus far obtained show that the method described above can be applied to more than the calibration of ENS detectors. Even with the comparatively poor statistics that we were able to obtain in our preliminary measurements the method gives interesting information on the details of the interaction of nucleons and nuclei at high energies. It may be expected that with better statistics other important features of these interactions will be revealed.* Experimental work is now being carried on in our laboratory with improved apparatus.

Finally, we wish to emphasize the fact that this method can be used not only for pions but for any other strongly interacting unstable particles. For example, it could be used to study short-lived high-energy particles produced in large accelerators at energies of the order of billions of electron-volts.

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*For example, the dependence of ρ on $\langle \epsilon \rangle$ can supply information concerning the creation of nucleon-antinucleon pairs at very high energies.

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HOLE CONDUCTIVITY IN SILVER BROMIDE

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(Presented by Academician A. F. Ioffe July 19, 1955)

A large number of papers have been devoted to the investigation of structural defects in silver halide salts. Both theoretical [1] and experimental data [2-6] have proved that Fraenkel defects predominate in silver bromide. However, many authors [4, 5, 7] have also assumed the existence of Schottky defects in AgBr in small concentrations. This assumption was required for the explanation of the mechanism of the latent image [8] and for the appearance of metallic dendrites during electrolysis of AgBr single crystals [9]. In both instances, the formation of silver particles seemed to be due to the movement of a certain quantity of anions by means of lattice vacancies. It was suggested that vacant anionic sites are able to capture electrons to form F-centers just as in the case of alkali halides. Thus a group of F-centers around silver ions would lead to the appearance of metallic ions in the crystal lattice [10].

P. Meiklär and E. Putseiko [11] state that the vacant anion sites needed for the formation of F-centers result directly both from photolysis and from thermal dissociation of the salt through removal of halogen atoms from the lattice. In support of this hypothesis, they present the absorption spectra of silver halides after irradiation or heating of the crystals. In AgBr an additional absorption band was found with a peak at 430-440 m μ . In AgCl, this same band is found at 370-380 m μ . Both bands are identified as due to F-centers on the basis of the fact that their position in the absorption spectrum obeys, in first approximation, the empirical formula of Mollwo ($\nu d^2 = \text{constant}$) which was established for F-band peaks in alkali halides.

Recently it has been found [12, 13] that the latent image and metallic silver dendrites are formed and developed not in the crystal lattice but along slip planes or at the interfaces of polyhedral blocks.

In this connection we thought it would be of interest to determine whether the migration of halogen atoms is possible in an AgBr lattice as assumed by Meiklär and Putseiko or whether the halogen is dissociated only at the surface and "holes" are formed in the lattice whose motion is not connected with removal of the halogen from lattice sites. This problem may be attacked by a study of the additive coloration of AgBr in bromine vapor and by a study of its electric conductivity.

For the purpose of investigating the additive coloration, we used AgBr single crystals. The coloration was produced in a glass ampoule. The crystal was in the upper half in the form of a small cylinder with platinum electrodes fastened to its ends with springs through a thin layer of graphite. The lower half contained pure bromine. The ampoule was evacuated to a pressure of $10^{-4} - 10^{-5}$ mm Hg as the bromine was cooled with dry ice, and then sealed. The ampoule was placed in a two-storied electric furnace divided by a heat-insulating partition. The upper and lower halves of the furnace were heated independently. Constancy of temperature in each half was maintained through the use of a contact mercury thermometer. The conductivity of the samples was measured with a 1000 cycle current at voltages from 1 to 10 v in a bridge. The results for the conductivity of AgBr in the absence of bromine are given in Figure 1 (curve 1) and can be expressed by the equation

$$\sigma = 1,02 \cdot 10^5 \exp(-18200/RT) \text{ ohm}^{-1} \cdot \text{cm}^{-1} \quad (1)$$

The electric conductivity of AgBr has the same temperature coefficient as in the data obtained by Koch and Wagner [14] (curve 2 in Figure 1), but with a somewhat smaller coefficient before the exponential.

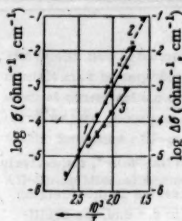


Figure 1.

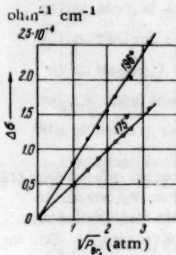


Figure 2.

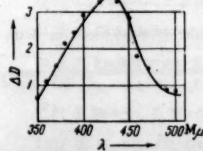


Figure 3.

The increase of electric conductivity of the crystal with increased halogen vapor pressure and increased temperature are exactly opposite.

The additional conductivity was calculated by taking the difference between the electric conductivity with and without the presence of bromine. Figure 2 gives the dependence of the hole conductivity of AgBr ($\Delta\sigma$) on the partial bromine pressure for two isotherms.* The origin for the coordinates chosen corresponds to the electric conductivity of pure AgBr in the absence of bromine. In all the cases investigated, the additional conductivity increases proportionately to the square root of the partial bromine pressure. The additional conductivity $\Delta\sigma$ increases as the temperature rises. Line 3 in Figure 1 shows the change of hole conductivity of AgBr for an isobar at 1 atmosphere as a function of the temperature (from 150 to 273°C) and may be represented by the equation

$$\Delta\sigma = 1.82 \cdot 10^3 \sqrt{P_{Br}} \exp(-13520/RT) \text{ ohm}^{-1} \text{ cm}^{-1} \text{ atm}^{1/2}. \quad (2)$$

Measurements were also taken of the absorption spectrum of AgBr after coloration by bromine vapor. These measurements were taken with an SF-4 spectrophotometer using a differential method. The standard of comparison was the same crystal without a halogen atmosphere. The results are given in Figure 3.

From the above data the following conclusions can be drawn:

1. When an AgBr crystal is heated in a bromine atmosphere, "holes" are formed. By-passing the intermediate states of the process (the dissociation of the bromine molecules and the adsorption of the bromine atoms on the crystal faces), the following equation can be given:



from which by the conditions of equilibrium there follows:

$$C_{Br_0} = \sqrt{K_1} \sqrt{P_{Br_2}}. \quad (4)$$

Here C_{Br_0} is the "hole" concentration in the crystal; K_1 is the equilibrium constant of the reaction (3). The formation of "holes" in additive coloration is a bimolecular process so that under conditions of isothermal equilibrium their concentration changes proportionately to the square root of the partial bromine pressure.

2. The positive "holes" and the vacant cation sites which are opposite in effective charge can interact with the formation of V_1 -centers:**



* The data for other temperatures is given in the form of circles in Figure 1 (line 3) for the 1 atmosphere isobar.

** Here and hereafter, a V_1 center is regarded as an association of one vacant cation site and a "hole". In their properties and in their absorption spectrum, V_1 -centers correspond to the V_F -centers in Seitz's terminology [16].

The concentration of V_1 -centers can be given by

$$C_{[Ag_{\square}^{+}Br_O]} = K_3 C_{Ag_{\square}^{+}} C_{Br_O}, \quad (6)$$

where $[Ag_{\square}^{+}Br_O]$ denotes a V_1 -center, and $Ag_{\square}^{+}$ and Br_O are the vacant cation site and the "hole", respectively.

As can be seen from (5) the formation of V_1 -centers is also a bimolecular process but in the direction opposite to the formation of "holes" (association instead of dissociation). Therefore, in the end, the equilibrium V_1 -center concentration changes at constant temperature proportionately to the partial pressure of bromine gas molecules. Combining (3) and (5), we obtain

$$Br_2 + Ag_{\square}^{+} \rightleftharpoons [Ag_{\square}^{+}Br_O] + Br_O, \quad (7)$$

whence we obtain

$$C_{[Ag_{\square}^{+}Br_O]} = K_3 P_{Br_2} \frac{C_{Ag_{\square}^{+}}}{C_{Br_O}} \quad (8)$$

The proportionality between the bromine pressure and the concentration of V-centers induced Seitz [16] to regard the latter as groups of two vacant cation sites and two "holes" and to call them V_2 -centers. In actuality, there are V_1 -centers in equilibrium with the bromine vapor if it is taken into consideration that during the process of additive coloration there is also a change of the concentration of vacant sites. Thus the concentration of V_1 -centers is dependent both on the partial halogen pressure and on the concentration of crystal structure defects (equations (6), (8) and (9)). In the formation of V_1 -centers and of "holes" in AgBr, neither atoms nor molecules of bromine are dissolved in the crystal lattice.

As follows from equation (4) the "hole" concentration is also dependent on the bromine pressure. With allowance for this change we obtain

$$C_{[Ag_{\square}^{+}Br_O]} = K_4 \sqrt{P_{Br_2}} C_{Ag_{\square}^{+}} \quad (9)$$

In this equation

$$K_4 = K_3 : \sqrt{K_1}, \quad K_4 \sqrt{P_{Br_2}} = K_3 C_{Br_O}.$$

3. Although the maximum concentration of V_1 -centers and of "holes" increases with higher bromine pressure, they cannot increase indefinitely. With increase in the number of "holes" there is a corresponding decrease of the concentration of free (not bound in V_1 -centers) vacant cation sites, so that their product remains approximately constant (for $T = \text{constant}$). The overall equilibrium of the concentration of vacant cation sites in the crystal is dependent only on the temperature and does not change during additive coloration. "Holes" arise at the surfaces of the crystal which are in contact with adsorbed bromine atoms.

The penetration of "holes" into the crystal is accompanied by the removal of an equivalent number of interstitial silver ions. Therefore, the stoichiometric excess of the halogen should be more correctly designated as a stoichiometric deficiency of metal atoms. Direct confirmation of this is given by the experiments of Uchida and Nakai [17] which show the possibility of obtaining V_1 -centers in KI and NaI by an electrochemical method—the injection of "holes" into the crystal from a pointed anode similar to the way in which F-centers appear in alkali halide crystals when electrons are injected from the cathode.

4. The additive coloration absorption band of AgBr (Figure 3) agrees in shape and spectral position with the additional band observed by Melikhar and Putseiko in silver bromide after irradiation or thermal dissociation. The same band was observed by Mollwo [18] in additively colored KBr on the long-wave side of the V-band. With high "hole" mobility and shallow electron-trapping levels in AgBr (1.1 eV [19]) the presence of F-centers in crystals with a stoichiometric excess of the halogen is very unlikely. The absorption bands in AgBr or KBr with a peak around 420 mμ belong to bromine molecules adsorbed on external or internal crystal surfaces.

The qualitative agreement of this band in AgBr with Mollwo's empirical formula for F-centers in alkali halide crystals must be regarded as purely accidental. To a large extent, the position of these bands agrees with the spectral characteristic curve for gaseous bromine [20]. Halogen atoms do not diffuse in the crystal lattice; therefore, the photolysis of AgBr cannot lead to the appearance of the vacant anion sites that are required for the formation of F-centers.

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THE RELATIONS BETWEEN CROSS SECTIONS FOR MULTIPLE PION PRODUCTION

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By the use of the hypothesis of isotopic invariance it is easy to obtain a number of relations between the cross sections for multiple production of pions from collisions of pions and nucleons with nucleons. In the vicinity of the threshold of these processes a number of supplementary relations hold good, which it is the purpose of the present note to establish.

Let us consider collisions of pions with nucleons (n) as a result of which a pion is transformed into two pions, i.e., the reaction

$$\pi + n \rightarrow \pi' + \pi'' + \pi''' \quad (1)$$

The initial state is a superposition of states with isotopic spin $T = \frac{1}{2}$ or $\frac{3}{2}$.

The wave functions of the final state corresponding to these values of the total spin T may be constructed in several ways out of the wave functions of the nucleon (isotopic spin $\frac{1}{2}$) and the wave functions of two pions with isotopic spin $\underline{1}$ equal to 0, 1, or 2.

Denoting by $A_{\underline{1}}^T$ the amplitude of the transition to a state with total isotopic spin T and isotopic spin of the system of pions $\underline{1}$, we have:

$$\begin{aligned} d\sigma_1(\pi^+ + p \rightarrow p + \pi^+ + \pi^0) &= \left| \frac{1}{\sqrt{10}} A_2^{1/2} - \frac{1}{\sqrt{2}} A_1^{1/2} \right|^2; \\ d\sigma_2(\pi^- + p \rightarrow p + \pi^- + \pi^0) &= \left| \frac{1}{\sqrt{10}} A_2^{1/2} - \frac{\sqrt{2}}{3} A_1^{1/2} - \frac{1}{3\sqrt{2}} A_1^{1/2} \right|^2; \\ d\sigma_3(\pi^- + p \rightarrow n + \pi^0 + \pi^0) &= \frac{2}{9} \left| A_0^{1/2} + \sqrt{\frac{2}{5}} A_2^{1/2} \right|^2; \\ d\sigma_4(\pi^- + p \rightarrow n + \pi^+ + \pi^-) &= \frac{2}{9} \left| A_0^{1/2} - \frac{1}{\sqrt{10}} A_2^{1/2} - \frac{1}{\sqrt{2}} A_1^{1/2} + \frac{1}{\sqrt{2}} A_1^{1/2} \right|^2; \\ d\sigma_5(\pi^+ + p \rightarrow n + \pi^+ + \pi^+) &= \frac{4}{5} \left| A_2^{1/2} \right|^2; \\ d\sigma_6(\pi^0 + p \rightarrow p + \pi^0 + \pi^0) &= \frac{1}{9} \left| A_0^{1/2} - 2\sqrt{\frac{2}{5}} A_2^{1/2} \right|^2; \\ d\sigma_7(\pi^0 + p \rightarrow p + \pi^+ + \pi^-) &= \frac{1}{9} \left| A_0^{1/2} + \sqrt{\frac{2}{5}} A_2^{1/2} - \frac{1}{\sqrt{2}} A_1^{1/2} - \sqrt{2} A_1^{1/2} \right|^2; \\ d\sigma_8(\pi^0 + p \rightarrow n + \pi^+ + \pi^0) &= \left| \frac{1}{\sqrt{5}} A_2^{1/2} - \frac{1}{3} A_1^{1/2} + \frac{1}{3} A_1^{1/2} \right|^2. \end{aligned} \quad (2)$$

We obtain herefrom relations between the total cross sections that agree with the relations obtained in [1]:

$$\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \sigma_5 = 2(\sigma_6 + \sigma_7 + \sigma_8); \quad (3)$$

$$\sigma_1 + \sigma_2 + \sigma_3 + 4(\sigma_4 + \sigma_5) = 2(\sigma_6 + \sigma_7 + \sigma_8). \quad (4)$$

The wave functions of the final states with isotopic spins $t = 0$ and $t = 2$ are symmetrical in the charge variables of the pions and are, consequently, symmetrical in the coordinates of the pions (pions obey Bose statistics). The wave function of a state with isotopic spin $t = 1$ is antisymmetrical in the charge variables of the pions and is therefore antisymmetrical in the coordinates. Near the threshold of reaction (1), the pions are produced in an S-state (we are considering their relative motion). Since the wave function of the final state must be symmetrical with respect to permutation of the pions, the production of two pions with $t = 1$ is forbidden in the vicinity of the threshold of reaction (1).

Assuming in (2) $A_1^{1/2} = A_2^{1/2} = 0$, we obtain the following relations between the differential cross sections near the threshold:

$$d\sigma_1(\pi^+ + p \rightarrow p + \pi^+ + \pi^0) = d\sigma_2(\pi^- + p \rightarrow p + \pi^- + \pi^0); \quad (5)$$

$$2d\sigma_1(\pi^+ + p \rightarrow p + \pi^+ + \pi^0) = d\sigma_8(\pi^0 + p \rightarrow n + \pi^+ + \pi^0); \quad (6)$$

$$8d\sigma_1(\pi^+ + p \rightarrow p + \pi^+ + \pi^0) = d\sigma_6(\pi^+ + p \rightarrow n + \pi^+ + \pi^+); \quad (7)$$

$$2d\sigma_7(\pi^0 + p \rightarrow p + \pi^+ + \pi^-) = d\sigma_3(\pi^- + p \rightarrow n + \pi^0 + \pi^0); \quad (8)$$

$$d\sigma_4(\pi^- + p \rightarrow n + \pi^+ + \pi^-) + d\sigma_8(\pi^0 + p \rightarrow n + \pi^+ + \pi^0) = \\ = d\sigma_6(\pi^0 + p \rightarrow p + \pi^0 + \pi^0) + d\sigma_7(\pi^0 + p \rightarrow p + \pi^+ + \pi^-). \quad (9)$$

Hence for the total cross sections we obtain

$$\sigma_1(\pi^+ + p \rightarrow p + \pi^+ + \pi^0) = \sigma_2(\pi^- + p \rightarrow p + \pi^- + \pi^0); \quad (10)$$

$$2\sigma_1(\pi^+ + p \rightarrow p + \pi^+ + \pi^0) = \sigma_8(\pi^0 + p \rightarrow n + \pi^+ + \pi^0); \quad (11)$$

$$4\sigma_1(\pi^+ + p \rightarrow p + \pi^+ + \pi^0) = \sigma_6(\pi^+ + p \rightarrow n + \pi^+ + \pi^+); \quad (12)$$

$$\sigma_7(\pi^0 + p \rightarrow p + \pi^+ + \pi^-) = \sigma_3(\pi^- + p \rightarrow n + \pi^0 + \pi^0); \quad (13)$$

$$\sigma_4(\pi^- + p \rightarrow n + \pi^+ + \pi^-) + \sigma_8(\pi^0 + p \rightarrow n + \pi^+ + \pi^0) = \\ = 2\sigma_6(\pi^0 + p \rightarrow p + \pi^0 + \pi^0) + \sigma_7(\pi^0 + p \rightarrow p + \pi^+ + \pi^-). \quad (14)$$

The same relations between the cross sections hold away from the threshold if the two pions in reaction (1) are produced with identical momentum $k_1 = k_2$.

In fact, in these cases also the wave function of the final state is symmetrical in the pion coordinates and, consequently, symmetrical in the charge variables. Therefore, the production of two pions with $t = 1$ is also forbidden for this case.

We shall now consider the transformation of a pion into two pions in collisions of pions with deuterons, i.e., the reaction

In the initial state the isotopic spin is unity. The wave function of the final state with $T = 1$ can be constructed either out of wave functions of the system of two nucleons with isotopic spin $t_1 = 1$ and wave functions of the system of two pions with isotopic spin $t_2 = 0, 1, 2$, or out of wave functions of the nucleons with $t_1 = 0$ and wave functions of the pions with $t_2 = 1$. Denoting by $B_{t_1 t_2}$ the amplitudes of the transition to states with isotopic spins t_1 and t_2 , we have:

$$\begin{aligned} d\sigma_1(\pi^+ + d \rightarrow p + n + \pi^+ + \pi^0) &= \frac{1}{4} \left| -\sqrt{\frac{3}{10}} B_{11} + B_{01} - \frac{1}{\sqrt{2}} B_{11} \right|^2; \\ d\sigma_2(\pi^+ + d \rightarrow p + p + \pi^0 + \pi^0) &= \frac{1}{3} \left| \frac{1}{\sqrt{5}} B_{11} - B_{10} \right|^2; \\ d\sigma_3(\pi^+ + d \rightarrow p + p + \pi^+ + \pi^-) &= \left| \frac{1}{\sqrt{60}} B_{11} + \frac{1}{2} B_{11} + \frac{1}{\sqrt{3}} B_{10} \right|^2; \\ d\sigma_4(\pi^+ + d \rightarrow n + n + \pi^+ + \pi^+) &= \frac{3}{5} |B_{11}|^2; \\ d\sigma_5(\pi^0 + d \rightarrow p + n + \pi^0 + \pi^0) &= \frac{1}{6} \left| \frac{2}{\sqrt{5}} B_{11} + B_{10} \right|^2; \\ d\sigma_6(\pi^0 + d \rightarrow p + n + \pi^+ + \pi^-) &= \frac{1}{6} \left| -\frac{1}{\sqrt{5}} B_{11} + B_{10} + \frac{\sqrt{6}}{2} B_{01} \right|^2; \\ d\sigma_7(\pi^0 + d \rightarrow n + n + \pi^+ + \pi^0) &= \frac{1}{2} \left| \sqrt{\frac{3}{10}} B_{11} + \frac{1}{\sqrt{2}} B_{11} \right|^2. \end{aligned} \quad (16)$$

Hence we obtain for the total cross sections the relations

$$\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 = \sigma_5 + \sigma_6 + 2\sigma_7; \quad (17)$$

$$2(\sigma_3 + \sigma_4) + \sigma_6 = \sigma_1 + 4\sigma_2 + 2\sigma_5 + \sigma_7, \quad (18)$$

which also agree with those obtained in [1].

Near the threshold of reaction (15) and also with the production in this reaction of pions with identical momenta ($\mathbf{k}_1 = \mathbf{k}_2$) $B_{111} = 0$.

In these instances the following relations hold between the differential cross sections:

$$2d\sigma_1(\pi^+ + d \rightarrow p + n + \pi^+ + \pi^0) = d\sigma_7(\pi^0 + d \rightarrow n + n + \pi^+ + \pi^0); \quad (19)$$

$$8d\sigma_1(\pi^+ + d \rightarrow p + n + \pi^+ + \pi^0) = d\sigma_4(\pi^+ + d \rightarrow n + n + \pi^+ + \pi^+); \quad (20)$$

$$2d\sigma_4(\pi^0 + d \rightarrow p + n + \pi^+ + \pi^-) = d\sigma_3(\pi^+ + d \rightarrow p + p + \pi^0 + \pi^0); \quad (21)$$

$$2d\sigma_1(\pi^+ + d \rightarrow p + n + \pi^+ + \pi^0) + d\sigma_3(\pi^+ + d \rightarrow p + p + \pi^+ + \pi^-) = d\sigma_5(\pi^0 + d \rightarrow p + n + \pi^0 + \pi^0) + d\sigma_6(\pi^0 + d \rightarrow p + n + \pi^+ + \pi^-), \quad (22)$$

and hence the relations between the total cross sections are

$$\sigma_1(\pi^+ + d \rightarrow p + n + \pi^+ + \pi^0) = \sigma_7(\pi^0 + d \rightarrow n + n + \pi^+ + \pi^0); \quad (23)$$

$$2\sigma_1(\pi^+ + d \rightarrow p + n + \pi^+ + \pi^0) = \sigma_4(\pi^+ + d \rightarrow n + n + \pi^+ + \pi^+); \quad (24)$$

$$\sigma_4(\pi^0 + d \rightarrow p + n + \pi^+ + \pi^-) = 2\sigma_3(\pi^+ + d \rightarrow p + p + \pi^0 + \pi^0); \quad (25)$$

$$2\sigma_1(\pi^+ + d \rightarrow p + n + \pi^+ + \pi^0) + 2\sigma_2(\pi^+ + d \rightarrow p + p + \pi^+ + \pi^-) = \\ = 2\sigma_3(\pi^0 + d \rightarrow p + n + \pi^0 + \pi^0) + \sigma_4(\pi^0 + d \rightarrow p + n + \pi^+ + \pi^-). \quad (26)$$

We note that in the above cases (near the threshold of the reaction or with identical momenta of the created pions) the nucleons in reaction (15) are produced only in the singlet state and that the reaction

$$\pi + d \rightarrow d + \pi' + \pi'', \quad (27)$$

i.e., the reaction without deuteron splitting, is forbidden.

Other processes can be investigated in a similar way. For example, for reactions close to the threshold of the creation of two pions in the collision of two nucleons similar relations hold between the differential cross sections:

$$d\sigma(p + p \rightarrow p + n + \pi^+ + \pi^0) = d\sigma(p + n \rightarrow p + p + \pi^- + \pi^0) = \\ = 1/2 d\sigma(p + p \rightarrow n + n + \pi^+ + \pi^+). \quad (28)$$

Hence, and also from the relation which is easily obtained by the method described in [1], we obtain for the total cross sections

$$\sigma(p + p \rightarrow p + n + \pi^+ + \pi^0) = 1/2 \sigma(p + p \rightarrow n + n + \pi^+ + \pi^+) = \\ = 2\sigma(p + n \rightarrow p + p + \pi^- + \pi^0) = \\ = 2\sigma(p + p \rightarrow p + p + \pi^0 + \pi^0) + 2\sigma(p + n \rightarrow p + n + \pi^0 + \pi^0) - \\ - \sigma(p + p \rightarrow p + p + \pi^+ + \pi^-) - \sigma(p + n \rightarrow p + n + \pi^+ + \pi^-). \quad (29)$$

We note also that the reaction $p + p \rightarrow d + \pi^+ + \pi^0$ is forbidden at the threshold. (This follows from the fact that the isotopic spin of the initial state is unity and the spin of the deuteron is zero. Consequently, two pions must be produced in a state with isotopic spin equal to unity, which is forbidden at the threshold). Moreover, the differential cross sections of the reactions $p + n \rightarrow d + \pi^0 + \pi^0$ and $p + n \rightarrow d + \pi^+ + \pi^-$ are equal at the threshold (and, consequently, the total cross section of the first reaction is one half as large as the total cross section of the second reaction).

For the transformation of one pion into three in collisions of a pion with a nucleon the only non-vanishing amplitudes near the threshold are $C_T^T = C_{1/2}^T, C_{3/2}^T$, where T is the total isotopic spin of the entire system and t is the isotopic spin of the system of three pions. For analogous reactions with deuterons the non-vanishing amplitudes are D_{1t_2}, D_{3t_2} , where t_2 is the isotopic spin of the system of three pions.

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ON INCREASING THE EFFICIENCY OF SEMICONDUCTOR THERMOCOUPLES

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N. V. Kolomoets and L. S. Stifbans

From A. F. Ioffe's theory of the power-production applications of semiconductor thermocouples [1] follows that the efficiency of the thermocouples when used in thermoelectric generators and thermoelectric refrigerators is proportional to the ratio of the mobility of the charge carriers to the thermal conductivity of the lattices in the arms of the thermocouple. The efficiency of a thermocouple attains its maximum value at a certain carrier concentration at which the thermoelectric force of each of the arms is approximately $\pm 200 \mu\text{V}/\text{degree}$.

Thus an increase of the efficiency of these thermoelectric instruments reduces to the solution of three basic problems:

1. The finding of materials with maximum ratio of carrier mobility to thermal conductivity of the lattice.
2. The creation in these materials of a carrier concentration which will ensure an optimal thermoelectric force of $\pm 200 \mu\text{V}/\text{degree}$.
3. The development of methods of further raising the ratio of the carrier mobility to the thermal conductivity in these materials.

The last problem may be solved as follows. Into the crystalline lattice of the thermocouple material we introduce another substance which crystallizes in a similar lattice and which has approximately the same lattice constant. Such binary (or pseudobinary, if the initial components are chemical compounds) systems usually form a continuous system of solid solutions or at least permit the mutual solution of the initial compounds within wide limits.

The distortions of the basic lattice due to the introduction of the admixture are relatively small (because of the isomorphism of the original materials) and are confined to the crystal regions in direct contact with the added atoms. Such distortions are quite efficient in the scattering of thermal oscillations whose wave lengths at ordinary temperatures do not much exceed the lattice constant. As a result, the thermal conductivity of the lattice is considerably reduced.

At the same time these admixtures, which on the average disturb the periodicity of the lattice very little, are not effective for the scattering of electron waves of longer wave length and thus cause an insignificant decrease of the carrier mobility.

Thus it is possible to achieve a considerable increase of the ratio of carrier mobility to thermal conductivity of the lattice.

This method has been tested practically on a number of solid semiconductors used as arms of thermocouples, and in all cases a considerable increase of the thermocouple efficiency was observed.

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RADIATION FROM A PLASMA ELECTRON IN UNIFORM MOTION IN A MAGNETIC FIELD*

A. A. Kolomenskii

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The uniform motion of an electron in a medium of dielectric constant ϵ is accompanied by a specific radiation of electromagnetic waves (Cerenkov effect [2]) if the following condition is satisfied:

$$n > \frac{1}{\beta}, \quad (1)$$

where $\beta = v/c$; v is the velocity of the electron, and n is the index of refraction, which in the case of anisotropic medium is given by

$$n = \sqrt{\epsilon}. \quad (2)$$

In the case of electron plasma, ϵ may be given by

$$\epsilon(\omega) = 1 - \frac{\omega_0^2}{\omega^2}, \quad (3)$$

where ω is the frequency of the propagating wave; ω_0 is the plasma frequency

$$\omega_0 = \sqrt{\frac{4\pi N e^2}{m}}; \quad (4)$$

N is the number of electrons per unit volume; and e and m are the charge and mass of the electron.

From (2) and (3) it follows that the phase velocity of wave propagation in a plasma always exceeds the speed of light because of which (1) cannot be satisfied and the Cerenkov effect cannot take place in a plasma, as is well known. V. I. Veksler pointed out that the Cerenkov effect may be possible in a plasma within a magnetic field. The present article discusses some results of a study of this phenomenon.

It is known that an electron plasma in a magnetic field behaves like a birefringent hypertropic crystal with its axis of symmetry in the direction of the magnetic field H . The refractive index n in such a plasma is a function of various variables $n = n_{\theta, e}(\omega, \theta, \omega_0, \omega_H)$ where θ is the angle between a given direction and the direction of H ; ω_H is the so-called gyroscopic frequency $\omega_H = eH/mc$, and the indexes o and e refer, respectively, to the ordinary and extraordinary waves (see, for example, [3]).

From general considerations of interference it follows that the condition for the Cerenkov effect also applies to the anisotropic and gyrotropic medium which we are investigating. At the same time an examination of the expression for $n_{\theta, e}$ shows that for any value of the magnetic field it is possible to find frequencies

* An investigation by the author of radiation from a plasma electron in a magnetic field was completed in 1951 [1].

which correspond to the fulfillment of (1). From these considerations it follows that the following well-known relation remains valid

$$\cos \vartheta = \frac{1}{n\beta} = \frac{\lambda}{\lambda_0}, \quad (5)$$

where ϑ is the angle between the direction of the electron velocity and the direction of emission $\mathbf{k}$ (the wave normal); λ is the wave length and λ_0 is the distance traversed by the electron per wave period. The electron radiates when moving at any angle to $\mathbf{H}$. For the sake of simplicity we shall confine ourselves here to the case of electron motion in the direction of $\mathbf{H}$, in which case $\vartheta = 0$. Substituting in (5) the formula $n = n(\omega, \theta, \omega_0, \omega_H)$ and solving the resulting equation for ϑ we obtain

$$\begin{aligned} \cos^2 \vartheta_{1,2} = & \frac{\omega^2}{2\beta^2 \{ (\omega^2 - \omega_0^2)\beta^2 - \omega_H^2 \omega^2 [\omega^2 \beta^2 + (1 - \beta^2) \omega_0^2] \}} \times \\ & \times \{ 2(\omega^2 - \omega_0^2)\beta^2 - \omega_H^2 [2\beta^2 \omega^2 + (1 - \beta^2) \omega_0^2] \pm \\ & \pm \omega_0^2 \omega_H \sqrt{4(\omega^2 - \omega_0^2)\beta^2 + \omega_H^2 (1 - \beta^2)^2} \}. \end{aligned} \quad (6)$$

The frequency regions ω corresponding to Cerenkov radiation are determined (for given values of ω_0 , ω_H and β) by the requirement of real values for $\vartheta_{1,2}$ in (6), i.e., by the condition

$$0 \leq \cos^2 \vartheta_{1,2}(\omega) \leq 1. \quad (7)$$

From (6) and (7) it follows that the frequency regions of the emitted ordinary (a) and extraordinary (b) waves may be found between the limits given by the inequalities

$$\text{a) } \omega < \omega_0; \quad \text{b) } \omega_0 < \omega < \sqrt{\omega_0^2 + \omega_H^2}, \quad (8)$$

i.e., these regions can never overlap. In comparatively weak magnetic fields $H < H_1$ only the extraordinary waves are emitted, but for $H > H_1$ the ordinary waves also, where

$$H_1 = \begin{cases} \frac{8\pi Nec\beta}{\sqrt{1 - \beta^2}}, & \text{if } \beta < \frac{1}{\sqrt{2}}; \\ \frac{4\pi Nec}{1 - \beta^2}, & \text{if } \beta > \frac{1}{\sqrt{2}}. \end{cases} \quad (9)$$

In the relativistic case ($\beta \sim 1$) only the extraordinary waves are emitted and in this case fill the entire range of (8b). We note that in the medium under consideration Cerenkov radiation is also present in the nonrelativistic case ($\beta \ll 1$).

The energy spectrum of the above radiation can be calculated in several ways. The general case of radiation by an electron in an anisotropic and gyrotropic medium, whose properties are described by the dielectric tensor

$$\epsilon_{ik} = \begin{pmatrix} \epsilon & -ie_1 & 0 \\ ie_1 & \epsilon & 0 \\ 0 & 0 & \epsilon_0 \end{pmatrix}, \quad (10)$$

have been examined by the author [1, 4], who obtained a formula for the radiation losses (per unit path length) dW_{rad}/dz .

In the case of a plasma in a magnetic field the components of the tensor ϵ_{ik} are [3]

$$\epsilon = \frac{\omega^2 - \omega_0^2 - \omega_H^2}{\omega^2 - \omega_H^2}, \quad \epsilon_0 = 1 - \frac{\omega_0^2}{\omega^2}, \quad \epsilon_1 = \frac{\omega_0^2 \omega_H}{\omega (\omega^2 - \omega_H^2)}, \quad (11)$$

and the expression for dW_{rad}/dz becomes

$$\frac{dW_{\text{rad}}}{dz} = \frac{e^2}{2c^3} \int \left| \frac{(\omega^2 - \omega_H^2)(1 - \beta^2) + \beta^2 \omega_0^2}{\beta^2 (\omega_0^2 + \omega_H^2 - \omega^2)} \left(1 \pm \frac{\omega_H [(\omega^2 - \omega_H^2)(1 - \beta^2)^2 + \beta^2 (3 - \beta^2) \omega_0^2]}{(\omega^2 - \omega_H^2)(1 - \beta^2) + \beta^2 \omega_0^2} \sqrt{(1 - \beta^2)^2 \omega_H^2 + 4\beta^2 (\omega^2 - \omega_0^2)} \right) \right| \omega d\omega, \quad (12)$$

where the integral is extended over the regions in which emission can occur.

This expression at the same time gives the energy spectrum of the radiation.

It is interesting that the application of a magnetic field to the plasma changes the nature of the energy losses of the moving electron. In the absence of the magnetic field the electron suffers only polarization losses corresponding to waves whose frequency is equal to the plasma frequency ω_0 . These waves, for which $\epsilon = 0$ (see [3]) are damped exponentially at a distance from the path of the electron. In a magnetic field there are no frequencies corresponding to simultaneous vanishing of the components of the dielectric tensor (11). Instead of polarization losses in this case there are Cerenkov losses so that (12) actually gives the total energy losses of the electron (we refer everywhere to losses by so-called distant collisions; see, for example, [5]).

A study of the loss integral (12) shows that it diverges at the upper limit of the extraordinary waves $\omega_{\text{lim}} = \sqrt{\omega_0^2 + \omega_H^2}$. This kind of divergence of the integral of Cerenkov losses is altogether peculiar to anisotropic (and gyrotropic) media as distinct from isotropic media, which has already been noted by the author [4, 6].

In order to comprehend the physical significance of the singularity in the region $\omega \approx \omega_{\text{lim}}$ we must turn our attention to the unusual behavior of the electromagnetic wave vectors in the given medium. Only such cases of wave propagation need be considered which satisfy the emission condition (5). This condition establishes a definite connection between the frequency ω and the direction of propagation, i.e., the angle ϑ [see (6)]. According to (6) the frequency ω_{lim} corresponds to the direction of emission perpendicular to the motion of the electron, i.e., $\vartheta = \pi/2$. By the use of the expression for ϵ_{ik} [see (10) and (11)], it can be shown that for $\vartheta = \pi/2$ and $\omega = \omega_{\text{lim}}$ the electric vector is linearly polarized and has the direction of the propagated wave. In other words, the electric field wave is here longitudinal.

In addition, the limiting case $\omega = \omega_{\text{lim}}$, $\vartheta = \pi/2$ according to (5) corresponds to the case $\lambda = 0$, i.e. to a vanishingly small wave length, where the results of the macroscopic theory cannot, strictly speaking, be used. Therefore, in the calculation of the integral of (12), the upper limit must be a value $\omega_{\text{max}} < \omega_{\text{lim}}$ corresponding, according to (5) and (6), to a finite value of $\lambda = \lambda_{\text{min}}$. This value must be chosen in accordance with the condition for applicability of the macroscopic theory. We then obtain for the losses (12) an expression of the same type as for the polarization losses in a plasma without a magnetic field (see, for example, [5]):

$$\frac{dW_{\text{rad}}}{dz} = \frac{e^2 \omega_0^3}{c^3} \ln \frac{\lambda_0}{\lambda_{\text{min}}}, \quad (13)$$

where λ_0 is the plasma wavelength and $\lambda_0 = 2\pi c / \omega_0$.

The above phenomenon has a number of additional special characteristics which are the subject of a more detailed paper by the author. Here it can only be noted that an electron plasma in a magnetic field is found under various natural circumstances such as, for example, the ionosphere in the earth's magnetic field

or the ionized atmospheres of the sun and stars which are affected by the corresponding magnetic fields, etc. Those regions of space are constantly being traversed by streams of charged cosmic particles of different velocities. Therefore the production of Cerenkov radiation of the type considered here with frequencies in the range given by (8) should evidently be a fairly widespread phenomenon. Variations of this radiation are also possible under laboratory conditions.

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A MICROTRON WITH WEAK PHASING

E. M. MOROZ

(Presented by Academician I. E. Tamm June 15, 1955)

The idea of a microtron, that is, of a resonance accelerator of relativistic particles operated with a constant magnetic field and constant frequency of electric field, was first advanced by V. I. Veksler [1] in 1944. The essence of the idea is that with each transit of a particle through the accelerating gap of the cavity resonator, it acquires energy equal to, or a multiple of, the quantity

$$\Delta E = \frac{e\lambda H}{2\pi}, \quad (1)$$

which we shall call a portion of resonance energy. Here e is the electron charge and λ is the wavelength in vacuum corresponding to the frequency of the electric field of the resonator. In successive transits through the resonator an equilibrium (or resonant) particle always encounters the same phase of the electric field $\varphi^{(r)}$ which may be obtained from the equation $2\pi V \cos \varphi^{(r)} / \lambda H = N$, where V is the potential difference applied to the accelerating gap and N is an integer. Hence we obtain the condition

$$a \equiv \frac{2\pi V}{\lambda H} > 1, \quad (2)$$

which leads to the necessity for using small values of H and thus small dimensions of the apparatus and limits the attainable energy. The phase equation of the microtron was first investigated by A. A. Kolomenskiĭ [2]. All of the theoretical and experimental work on the microtron which appeared after Veksler's paper was concerned with the above-mentioned scheme of acceleration [2-6]. In the present paper we are concerned with the existence of other accelerating schemes in which, in particular, the limitation (2) is not in effect.

It is possible to require that an accelerated particle receive one or more portions of resonance energy (1) not in one transit of the accelerating gap, but as the result of a cycle consisting of a few (1) successive transits. Of course, the particle will not then always encounter exactly the same phase of the electric field, but it is possible to require that after completing a cycle the particle should during the next cycle pass through the same sequence of phases. Under such an accelerating regime the microtron particle may encounter not only phases corresponding to a decrease of the electric field but also phases corresponding to its increase (unstable in the ordinary microtron) as well as phases corresponding to a decelerating field. Thus, it is possible to bring about an alternation of phasing and dephasing similar to the alternation of focusing and defocusing activity in strong focusing accelerators and through the choice of a suitable mode of operation obtain weak phasing in the microtron. Such schemes of operation may also be established by the simultaneous use in the microtron of two or more different frequencies of accelerating voltage.

In the present paper we shall deduce the equations of weakened modes of operation with a single accelerating frequency and examine one of these regimes.

In any resonance accelerator the phase φ_{q+1} of the electric field at the instant of the $(q+1)$ st transit of the particle through the accelerating gap is related to the phase φ_q of the high frequency field during the preceding q -th transit by the set of difference equations

$$\varphi_{q+1} = \varphi_q + \psi_q; \quad \psi_{q+1} = \psi_q + 2\pi a \cos \varphi_{q+1}, \quad (3)$$

whose summation gives

$$\varphi_q = \varphi_1 + \sum_{p=1}^{q-1} \psi_p; \quad \psi_p = \psi_0 + 2\pi a \sum_{h=1}^p \cos \varphi_h. \quad (4)$$

Here

$$\psi_0 = 2\pi \frac{2\pi E_0}{e\lambda H} \left(1 + \frac{eV_1}{E_0} \right);$$

E_0 is the rest energy, and eV_1 is the kinetic energy of the injected particle.

We require that the following resonance conditions be satisfied:

$$\psi_0^{(r)} = 2\pi P; \quad a \sum_{h=1}^l \cos \varphi_h^{(r)} = N; \quad (5)$$

$$\sum_{p=1}^l \psi_p^{(r)} = 2\pi M, \quad (6)$$

where P , N and M are integers. N is the number of portions of resonance energy (1) acquired by the particle during one accelerating cycle. A cycle consists of l successive passages of the accelerated particle through the resonator. The number P is the ratio of the frequency of the electric field to the rotational frequency of the injected particle in the magnetic field H . The upper index r indicates that the relations (5) and (6) refer to the resonance (equilibrium) values of φ and ψ . By substituting (5) and (6) in (3) and (4) and summing, it is not difficult to show that an accelerating regime exists which is characterized by the following periodicity relations:

$$\varphi_{m_l+n+1}^{(r)} = \varphi_{n+1}^{(r)} + 2\pi m \left[M + N \left(n + l \frac{m-1}{2} \right) \right]; \quad \psi_{m_l+n+1}^{(r)} = \psi_{n+1}^{(r)} + 2\pi m N,$$

where m and n are integers with $n < l$. Similarly it is possible to demonstrate the symmetry of the resonance phases which consists of the fact that for any integer $s < l$,

$$\varphi_{l-s}^{(r)} = \varphi_{s+1}^{(r)} + 2\pi [M - (s+1)N - (2s+1)P]; \quad \psi_{l-s}^{(r)} = -\psi_{s+1}^{(r)} + 2\pi (2P + N). \quad (7)$$

Setting in (7) $s = l-1$ and $s = 0$ we obtain two relations between $\varphi_1^{(r)}$ and $\varphi_l^{(r)}$ from which it follows that the integers M , N and P cannot be chosen independently:

$$M = Pl + \frac{l+1}{2} N. \quad (8)$$

If this relation is not satisfied, conditions (5) and (6) are incompatible, and if one of them is satisfied, (6) is a consequence of (5). From (8) it can be seen that if l is even, N must also be even. Thus there remains one equation for the resonance phases of (5):

$$a \sum_{h=1}^{\frac{l}{2}} \cos \varphi_h^{(r)} = \frac{N}{2} \quad \text{or} \quad 2a \sum_{h=1}^{\frac{l-1}{2}} \cos \varphi_h^{(r)} + a \cos \varphi_{\frac{l+1}{2}}^{(r)} = N, \quad (9)$$

respectively, for even and odd values of $\underline{l}$.

We shall now consider the stability of the solutions of (9). The linearized equations (3) become

$$\alpha_{q+1} = \alpha_q + \beta_q; \quad \beta_{q+1} = -S_{q+1}\alpha_q + (1 - S_{q+1})\beta_q, \quad (10)$$

where α_q and β_q are the small differences of φ_q and ψ_q from their equilibrium values: $S_q = 2\pi a \sin \varphi_q^{(r)}$.

By using (10) $\underline{l}$ times and making use of the symmetry property (7), it is possible to obtain expressions for $\alpha_{\underline{l}+q}$ and $\beta_{\underline{l}+q}$ in terms of α_q and β_q in the form of a set of difference equations with constant coefficients.

The stability condition for the motion described by this set of difference equations is $0 < S_1 < 4$ where $\underline{l} = 1$ and $\underline{l} = 2$, i.e., for the usual operating conditions of the microtron: $0 < 3(2S_1 + S_2) - 2S_1(S_1 + 2S_2) + S_1^2 S_2 < 4$ for $\underline{l} = 3$, etc.

Substituting the values of S_q obtained from the solution of (9), we find the region of stable resonance phases.

Table 1 gives the results of these calculations for regions of stable phases with $N = 1$, $\underline{l} = 3$ and, for comparison, the stable phase region for the usual microtron regime $N = 1$, $\underline{l} = 1$.

Table 1

Quantity	$\underline{l} = 1$	$\underline{l} = 3$
Midpoint of the stable phase region $\frac{\omega}{\varphi_1}^{(r)}$	0.28	0.20; 0.64; -1.106
Microtron constant $a(\varphi_1^{(r)})$	1.04	0.66; 0.75; 0.56
Phase width of the region $\Delta\varphi_1^{(r)}$	0.57	0.09; 0.05; 0.012
Amplitude width Δa	0.18	0.003; 0.023; 0.001

The change from the usual to the weakened mode of operation with $\underline{l} = 3$ enables us to decrease the amplitude or the frequency of the electric field of the resonator or to increase the magnetic field strength 1.4-1.7 times. Of course, this raises the stability requirements for λ , V and H .

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AN ANALYSIS OF SOME ANOMALOUS θ^0 DECAYS

E. Fridlender*

(Presented by Academician D. V. Skobel'tsyn November 11, 1955)

I. Introduction. Since the first observation of a θ^0 -particle [1] many investigators working in this field (see, for example, the reports of the Bagneres de Bigorre and Padua Congresses) have shown that its mass is close to the mass of a π meson ($975 m_e$). However, from time to time, reports appear about an "anomalous" θ^0 decay characterized in the majority of cases by a smaller value of Q than the usually accepted value of ~ 210 Mev.

In the course of a routine scanning of Ilford G-5 plates exposed to the action of cosmic rays at 2867 m an event** [2] was detected which provided grounds for a serious belief in the existence of a "light" θ^0 particle with a mass of about $600 m_e$. This induced us [3] to make a statistical analysis of θ^0 decays by means of which it might be possible to clarify certain general features of the "anomalous" decay (if it exists). This method will be described in part III.

II. "Anomalous θ^0 Decay." The angle between the prongs of the fork is 152° . From a point located in the central portion of the G-5 emulsion (400μ) the tracks of two particles emerge (designated by A and B). There is no visible recoil track.

Particle A is stopped in the emulsion after 540μ and evidently produces a σ -star (emission of a very slow proton). Its mass, as determined from range and scattering is $271 \pm 79 m_e$. Particle B escaped from the emulsion after traversing 2100μ . Its mass, as determined by scattering and grain density, was $260 \pm 41 m_e$.

Thus both particles are most likely pions. The change of ionization along the tracks shows that the particles were moving in opposite directions. The θ^0 mass was $595 \pm 36 m_e$ and, consequently, $Q = 23 \pm 18$ Mev. The emulsion was carefully checked in the region of the disintegration for anomalous ionization and systematic track distortions. Neither was found. Elementary collision dynamics eliminates the possibility of a "two-meson shower" produced by a fast neutron. All of these facts lead to the conclusion that the "anomalous" value of Q must be regarded as real.

III. Statistical Analysis of θ^0 Decays. We first set up a criterion for the rapid elimination of all decays except θ^0 decays. If p_T is the transverse momentum of the decay products, on a curve showing the dependence of p_T on Q all points corresponding to θ^0 decays should lie below the line

$$(p_T)_{\max} = \frac{Q}{2} \left(1 + \frac{4\mu}{Q} \right)^{1/2}, \quad (1)$$

where μ is the rest mass of a pion.

The calculation of the most probable value of the mass [4] of the θ^0 particle is obtained as follows. Let n measurements of p_T be taken; then their relative probabilities w_i can be calculated for the given value

* German spelling: E. Friedländer.

** Measurements of the particles were performed in the Structure of Matter of Laboratory of the Bucharest University; a preliminary report was presented at a scientific meeting of the university on June 21, 1953.

of $\vec{p}^*$ ($\vec{p}^*$ is the momentum of the decay products in the center-of-mass system). We define the distribution function L of the transverse momentum probabilities as

$$L = \prod_{i=1}^n w_i(p_{Ti}, p^*). \quad (2)$$

In [4] it is shown that if the probability density of p^* is a δ -function

$$F(p^*) dp^* = \delta(p^* - p_k^*) dp^*. \quad (3)$$

L has a sharp maximum at $p^* = p_k^*$.

In the absence of fluctuations the probability w_i is given by

$$w_i = \left(\frac{p_{Ti}}{p^*}\right) \left[1 - \left(\frac{p_{Ti}}{p^*}\right)^2\right]^{-1/2}$$

As was shown by Barker [5], the fluctuations can be taken into consideration and the values of w_i can be calculated by graphic integration.

IV. Comparison with Experiment. Preliminary calculations were carried out for 64 θ^0 -decays [2, 5, 14] selected in accordance with the criterion in (1). The histograms of p_T reveal two sharp peaks whose shape is similar to those found on the calculated curve of probabilities w_i . Then with the aid of the theoretical curve we calculated the probability distribution function L .

The curve of L showed two very sharp peaks at $595 \pm 10 m_e$ and $973 \pm 25 m_e$, in good agreement with directly observed values. Thus it is probable that a certain (small) fraction of θ^0 particles (θ_2^0) have a mass which is considerably smaller than the mass of ordinary θ^0 particles (θ_1^0).

The θ_2^0 lifetime is apparently greater (~ 10 times) than the θ_1^0 lifetime. This makes it possible to determine the origin of the θ_2^0 . The θ_2^0 may possibly be the neutral decay product of some unstable heavy particle such as a χ meson (see also [15]).

We are now using the same method to investigate the most anomalous θ^0 decays. The very first results indicate that the method of evaluating the maximum probability enables us to proceed towards a solution of the difficult problem of determining the masses of the various θ^0 .

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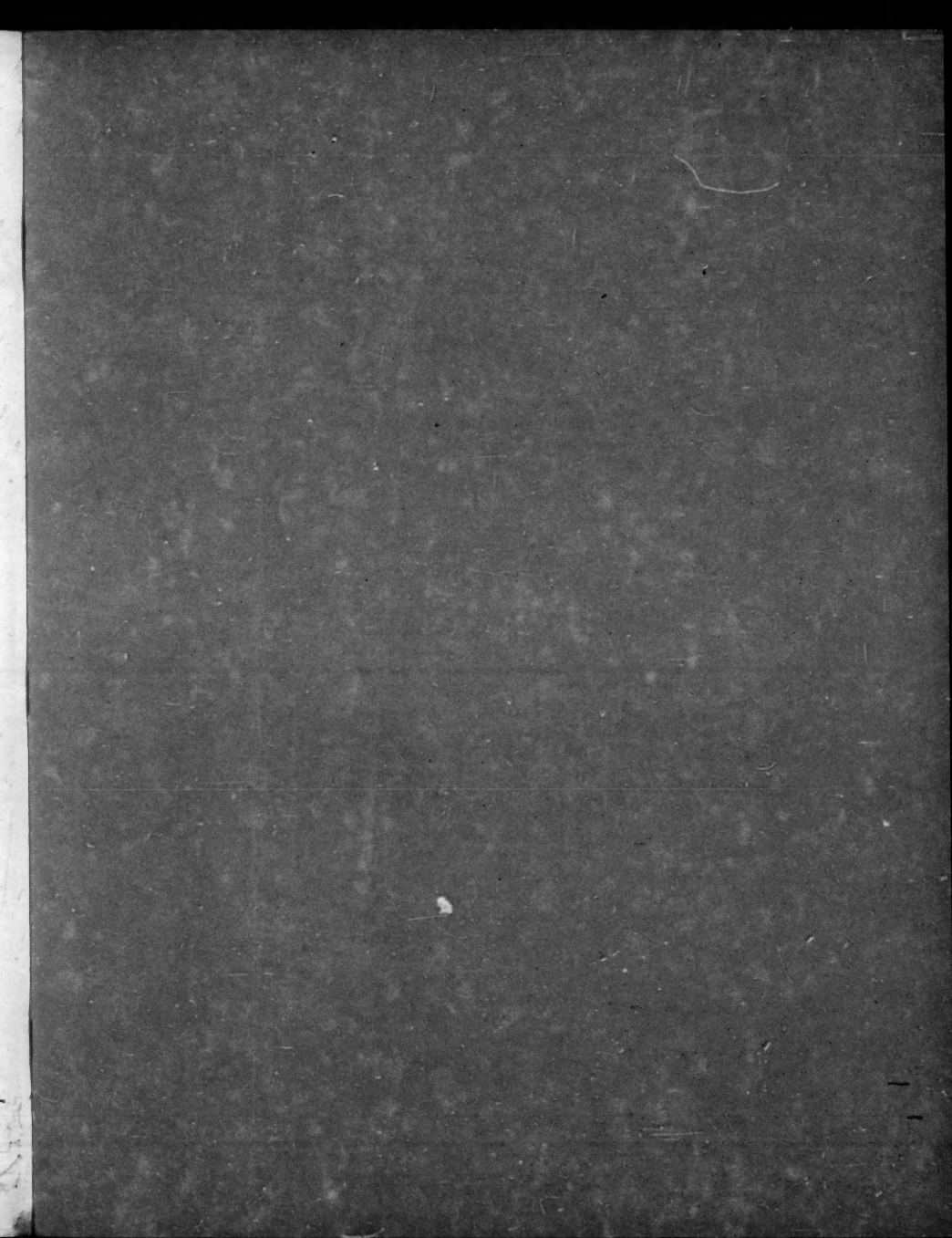
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